Curriculum Vitae

Personal Information

Family name: Velichkov   First name: Bozhidar   Web page: www.velichkov.it
Date of birth: 3 Feb 1985   e-mail: bozhidar.velichkov@unipi.it

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Positions and cursus

Since 1 June 2020 I am Full Professor (Professore Ordinario) at Università di Pisa (Italy).

2019 – 2020. I was Associate Professor (Professore Associato) at
Università degli Studi di Napoli Federico II (Naples, Italy).

2014 – 2019. I was Assistant Professor (Maître de Conférences) at
Laboratoire Jean Kuntzmann - Université Grenoble Alpes (Grenoble, France).

2014 – 2014. I spent six months as Post-doc in Shape Optimization at Università di Pisa .

2010 – 2013. PhD at Scuola Normale Superiore and, since 2012, also at Université de Savoie with advisors Giuseppe Buttazzo and Dorin Bucur; discussion: 8 Nov 2013 in Pisa, 70/70 cum Laude.

2005 – 2010. I was *Student in Mathematics* at SNS and I graduated with 70/70 cum Laude in 2010; as every SNS student, I was also a student at the University of Pisa:
- 2008 – 2010. Master in Mathematics - Università di Pisa (110/110 cum Laude);
- 2005 – 2008. Bachelor in Mathematics - Università di Pisa (110/110 cum Laude).

Honors and awards

2025. ERC Consolidator Grant
2024. Fubini Prize 2024.
2020. Book Prize UMI for my book *"Regularity of the one-phase free boundaries"* (Springer, 2023).
2019. ERC Starting Grant.
2013. My PhD Thesis *"Existence and Regularity Result for Some Shape Optimization Problems"* was selected for publication in Edizioni della Normale (Springer, 2015).

PhD students

Nuno Carneiro (since 2025; Università di Pisa, FCT);
Maria Antonietta Palladino (since 2024; Scuola Normale Superiore);
Federico Lai (since 2024; Università di Pisa);
Filippo Paiano (since 2023; Università di Pisa);
Matteo Carducci (since 2023; Scuola Normale Superiore);
Lorenzo Ferreri (since 2022; Scuola Normale Superiore);
François Générau (2017-2020; Université Grenoble Alpes); co-supervised with Edouard Oudet;
Baptiste Trey (2016-2020; Université Grenoble Alpes); co-supervised with Emmanuel Russ.

Post-docs

Giulia Bevilacqua (1 Sept 2022 – 31 Jan 2026);
Carlo Gasparetto (1 April 2023 – 30 April 2025);
Luca Benatti (1 Feb 2023 – 31 Jan 2024; co-supervised with Alessandra Pluda);
Roberto Ognibene (1 Feb 2022 – 31 March 2025);
Joseph Feneuil (1 Sept 2021 – 9 July 2022);
Giorgio Tortone (1 March 2021 – 28 Feb 2025).

Selected results

Logarithmic epiperimetric inequalities for the obstacle and the thin-obstacle problems

[CSV1] M. Colombo, L. Spolaor, B. Velichkov. *A logarithmic epiperimetric inequality for the obstacle problem.* **Geom. Funct. Anal.** 28 (4) (2018), 1029–1061.

[CSV2] M. Colombo, L. Spolaor, B. Velichkov. *Direct epiperimetric inequalities for the thin obstacle problem and applications.* **Comm. Pure. Appl. Math.** 73 (2) (2020), 384–420.

Regularity theory for one-phase free boundary problems

[FTV] L. Ferreri, G. Tortone, B. Velichkov. *A capillarity one-phase Bernoulli free boundary problem.* Preprint ArXiv (2023).

[FV] L. Ferreri, B. Velichkov. *Regularity for one-phase Bernoulli problems with discontinuous weights and applications.* **Trans. Amer. Math. Soc.** (2024), to appear.

[ESV] M. Engelstein, L. Spolaor, B. Velichkov. *Uniqueness of the blow-up at isolated singularities for the Alt-Caffarelli functional.* **Duke Math. J.** 169 (8) (2020), 1541–1601.

Regularity theory for two-phase free boundary problems

[SV] L. Spolaor, B. Velichkov. *An epiperimetric inequality for the regularity of some free boundary problems: the 2-dimensional case.* **Comm. Pure. Appl. Math.** 72 (2) (2018), 375–421.

[DSV1] G. De Philippis, L. Spolaor, B. Velichkov. *Regularity of the free boundary for the two-phase Bernoulli problem.* **Invent. Math.** 225 (2021), 347–394.

[DSV2] G. De Philippis, L. Spolaor, B. Velichkov. *(Quasi-)conformal methods in two-dimensional free boundary problems.* **J. Eur. Math. Soc.** (2024), doi: 10.4171/JEMS/1435

Regularity theory for vectorial Bernoulli problems and free boundary systems

[MTV1] D. Mazzoleni, S. Terracini, B. Velichkov. *Regularity of the optimal sets for some spectral functionals.* **Geom. Funct. Anal.** 27 (2017), 373–426.

[MTV2] D. Mazzoleni, S. Terracini, B. Velichkov. *Regularity of the free boundary for the vectorial Bernoulli problem.* **Anal. PDE** 13 (3) (2020), 741–764.

[MTV3] F. Maiale, G. Tortone, B. Velichkov. *Epsilon-regularity for the solutions of a free boundary system.* **Rev. Mat. Iberoam.** 39 (5) (2023), 1947–1972.

Regularity of optimal shapes

[BMMTV] G.Buttazzo, F. Maiale, D. Mazzoleni, G. Tortone, B. Velichkov. *Regularity of the optimal sets for a class of integral shape functionals.* **Arch. Rat. Mech. Anal.** (2024), to appear.

[MTV4] D. Mazzoleni, B. Trey, B. Velichkov. *Regularity of the optimal sets for the second Dirichlet eigenvalue.* **Ann. Inst. H. Poincaré Anal. Non Linéaire** 39 (3) (2022), 529–573.

[RTV] E. Russ, B. Trey, B. Velichkov. *Existence and regularity of optimal shapes for elliptic operators with drift.* **Calc. Var. PDE** 58 (2019).

[BMPV] D. Bucur, D. Mazzoleni, A. Pratelli, B. Velichkov. *Lipschitz regularity of the eigenfunctions on optimal domains.* **Arch. Rat. Mech. Anal.** 216 (2015), 117–151.

Regularity theory for optimal partition problems

[OV2] R. Ognibene, B. Velichkov. Structure of the free interfaces near triple junction singularities in harmonic maps and optimal partition problems. Preprint ArXiv (2024).

[OV1] R. Ognibene, B. Velichkov. Boundary regularity of the free interface in spectral optimal partition problems. Preprint ArXiv (2024).

Free boundary minimal surfaces

[BSV] G. Bevilacqua, S. Stuvard, B. Velichkov. Classical solutions to the soap film capillarity problem for plane boundaries. **Math. Ann.** 392 (2025), 4607–4659.

Mini courses

- "Free boundary regularity for the one-phase Bernoulli problem" (6 hours).
  Summer school "Free boundary problems and related topics" (ETH Zürich, 2022).

- "Regularity of the one-phase free boundaries" (6 hours).
  Summer school "Shape optimization, control and inverse problems for PDEs" (Naples, 2019).

Selected talks

- "Regularity up to the boundary for optimal partition problems"
  Workshop Calculus of Variations (Oberwolfach, 2024).

- "On the fine structure of the two-phase free boundaries"
  Workshop Partial Differentials Equations (Oberwolfach, 2023).

- "Free boundary clusters with two phases" MSRI Workshop "Regularity Theory for Minimal Surfaces and Mean Curvature Flow" - online (22/3/2022).

- "An epsilon-regularity theorem for the solutions of a vectorial free boundary system"
  Workshop Partial Differentials Equations (Oberwolfach, 2021).

- "Vectorial free boundary problems and regularity of the optimal sets for the eigenvalues of the Dirichlet Laplacian". One world PDE Seminar - online (2/3/2021).

- "Regularity of the two-phase free boundaries."
  Workshop Calculus of Variations (Oberwolfach, 2020).

- "Regularity of the two-phase free boundaries."
  XXX Convegno Nazionale di Calcolo delle Variazioni (Levico Terme, 2020).

- "On the logarithmic epiperimetric inequality."
  Partial Differentials Equations (Oberwolfach, 2019).

- "On the logarithmic epiperimetric inequality."
  XXIX Convegno Nazionale di Calcolo delle Variazioni (Levico Terme, 2019).

- Approche variationnelle à la régularité des frontières libres singulières.
  Laboratoire Jacques-Louis Lions (05/02/2018).

- "Variational approach to the regularity of the singular free boundaries."
  Seminar at ETH Zürich, 13/03/2018.

- "Recent results on the regularity of the free boundary of the obstacle problem."
  Calculus of Variations at Paris-Diderot (Paris, 2018).

- "Regularity of the free boundaries around isolated singularities."
  Seminar at Université Paris Sud - Orsay, 26/01/2018.

- "Regularity of the optimal sets for spectral functionals."
  Seminar at Max Planck Institut Leipzig, 13/05/2016.

- "Regularity of the optimal sets for spectral functionals."
  Seminar at Universität Zürich, 13/04/2016.Séminaire Parisien d'Optimisation, 6/10/2014.

Organization of workshops and conferences

Regularity Theory for Free Boundary and Geometric Variational Problems – one week conferences; 21 speakers per event; 5 editions: 2021 (Levico), 2022 (Pisa), 2023 (Levico), 2024 (Levico), 2025 (Cervia); jointly organized with Giulia Bevilacqua, Roberto Ognibene, Luca Spolaor.

Calculus of Variations and Free Boundary Problems – one day workshops; two in 2025 (Pisa); three in 2024 (Pisa); two in 2023 (Pisa); one in 2022 (Pisa), 2019 (Napoli), 2018 (Grenoble), 2017 (Grenoble); a complete list can be found here: http://www.velichkov.it/events.html

Projects

PI of an ERC Starting Grant project (2020-2025). PI of the project ERC Starting Grant "VAREG - Variational approach to the regularity of the free boundaries" (project number: 853404; duration: 66 months; volume 1,330 kE; starting date: 1 June 2020; host institution: Università di Pisa; web page: http://www.velichkov.it/vareg.html).

Local Coordinator of a national PRIN project (2023-2025). Local Coordinator (for Università di Pisa) of the project PRIN 2022 "NO^3 - Nodal Optimization, NOnlinear elliptic equations, NOnlocal geometric problems, with a focus on regularity" financed by MIUR (volume 200kE; duration: 24 months; PI: Nicola Soave).

PI of a project at University of Pisa (2022-2024). PI of the project PRA "GeoDom - Geometric evolution problems and PDEs on variable domains" financed by the University of Pisa (duration: 24 months; volume 50 kE; web page: http://www.velichkov.it/geodom.html).

Local Coordinator of a national ANR project (2018-2019). Local Coordinator (for Laboratoire Jean Kuntzmann – Université Grenoble Alpes) of the project ANR "ShapO - Shape Optimization" financed by the French National Research Agency - ANR (duration: 48 months; starting date: 10/2018; volume 300 kE; PI: Jimmy Lamboley).

PI of a project at Université Grenoble Alpes (2015-2016). PI of "VariForm - Méthodes Variationnelles en Optimisation de Formes" (Université Grenoble Alpes, 24 months, 15 kE).

Participation to other national projects. Member of the projects ANR "Geospec - Geometry and Spectral Optimization" (2016-2020) and ANR "CoMeDiC - Convergent Metrics for Digital Calculus" (2015-2020) financed by the French National Research Agency - ANR.

Selection committees and administration

PhD school in Mathematics (University of Pisa).

Since 11/2022 I am deputy coordinator of the PhD school in Mathematics at the Department of Mathematics, University of Pisa, appointed by the coordinator Roberto Frigerio.

In 2025 I was member (with Cecilia Pagliantini and Michele D'Adderio) of the selection committee for the entrance exam of the PhD school in Mathematics for the academic year 2025/2026.

In 2024 I was member (with Carlo Petronio and Cecilia Pagliantini) of the selection committee for the entrance exam of the PhD school in Mathematics for the academic year 2024/2025.

Selection committees at Scuola Normale Superiore.

In Sept 2020 and Sept 2022 I was member of the evaluation committees for the entrance exams (for the 1st and 4th years) at Scuola Normale Superiore respectively for the academic years 2020/2021 (chair: Franco Flandoli) and 2022/2023 (chair: Angelo Vistoli).

Referee of PhD thesis.

2024. Federico Franceschini (ETH Zürich; advisors: Alessio Figalli and Joaquim Serra);
2024. Clara Torres Latorre (Universitat de Barcelona; advisor: Xavier Ros-Oton);
2018. Harish Shrivastava (Università di Pisa; advisor: Giuseppe Buttazzo);

Selection committees for permanent positions.

2022. Call for Associate Professor (Professore Associato) at Università di Torino.

2021. Call for Full Professor (Professore Ordinario) at Università di Pisa.

2021. Call for a Tenure-Track (RTDB) position at Università di Torino.

List of publications

Preprints (for the complete list check my ArXiv page)

[P11] G. Siclari, B. Velichkov. On the blow-up of the vectorial Bernoulli free boundary problem. arXiv:2602.00741

[P10] C. Gasparetto, F. Paiano, B. Velichkov. *Regularity for free boundary surfaces minimizing degenerate area functionals.* arXiv:2503.02535.

[P9] A. Pacati, G. Tortone, B. Velichkov. *Some remarks on singular capillary cones with free boundary.* arXiv:2502.07697.

[P8] M. Carducci, B. Velichkov. *Existence and regularity in the fully nonlinear one-phase free boundary problem.* arXiv:2501.12101.

[P7] R. Ognibene, B. Velichkov. Structure of the free interfaces near triple junction singularities in harmonic maps and optimal partition problems. arXiv:2412.00781.

[P6] L. Ferreri, L. Spolaor, B. Velichkov. Unique continuation for nonlinear variational problems. arXiv:2408.00405.

[P5] L. Ferreri, L. Spolaor, B. Velichkov. On the boundary branching set of the one-phase problem. arXiv:2407.15230.

[P4] L. Ferreri, L. Spolaor, B. Velichkov. On the fine structure of the solutions to nonlinear thin two-membrane problems in 2D. arXiv:2405.05799.

[P3] R. Ognibene, B. Velichkov. Boundary regularity of the free interface in spectral optimal partition problems. arXiv:2404.05698.

[P2] L. Ferreri, G. Tortone, B. Velichkov. A capillarity one-phase Bernoulli free boundary problem. arXiv:2310.14309.

[P1] G. De Philippis, M. Engelstein, L. Spolaor, B. Velichkov : *Rectifiability and almost everywhere uniqueness of the blow-up for the vectorial Bernoulli free boundaries.* arXiv:2107.12485.

Journal articles

[A54] G. Tortone, B. Velichkov. *Vectorial Bernoulli problems and free boundary systems.*
La Matematica Volume 5, article number 16, (2026); special issue for Sandro Salsa.

[A53] R. Ognibene, B. Velichkov. *A survey on the optimal partition problem.*
La Matematica Volume 5, article number 6, (2026); special issue for Sandro Salsa.

[A52] M. Carducci, B. Velichkov. *An epiperimetric inequality for odd frequencies in the thin obstacle problem.* **J. Funct. Anal.** 289, No. 10, Article ID 111115, 31 p. (2025).

[A51] G. Bevilacqua, S. Stuvard, B. Velichkov. Classical solutions to the soap film capillarity problem for plane boundaries. **Math. Ann.** 392, No. 4, 4607–4659 (2025).

[A50] L. Ferreri, B. Velichkov. *A one-sided two phase Bernoulli free boundary problem.*
J. Math. Pures Appl. 195, Article ID 103659, 42 p. (2025).

[A49] S. Guarino Lo Bianco, D. A. La Manna, B. Velichkov. *Free boundary cluster with Robin condition on the transmission Interface.* **Calc. Var. PDE** 64 (2024), art. 4.

[A48] L. Ferreri, B. Velichkov. *Regularity for one-phase Bernoulli problems with discontinuous weights and applications.* **Trans. Amer. Math. Soc.** 377 (11) (2024), 7847–7876.

[A47] D. Mazzoleni, G. Tortone, B. Velichkov. *On the dimension of the singular set in optimization problems with measure constraint.* **J. Conv. Anal.** 31, No. 2, 689–708 (2024) (special issue for Giuseppe Buttazzo).

[A46] G. Buttazzo, F. Maiale, D. Mazzoleni, G. Tortone, B. Velichkov. *Regularity of the optimal sets for a class of integral shape functionals.* **Arch. Rat. Mech. Anal.** 248 (2024), article n. 44.

[A45] G. De Philippis, L. Spolaor, B. Velichkov. *(Quasi-)conformal methods in two-dimensional free boundary problems.* **J. Eur. Math. Soc.** 27, No. 8, 3369–3406 (2025).

[A44] N. Edelen, L. Spolaor, B. Velichkov. *The symmetric (log-)epiperimetric inequality and a decay-growth estimate.* **Calc. Var. PDE** 63, No. 1, Paper No. 2, 29 p. (2024).

[A43] F. Maiale, G. Tortone, B. Velichkov : *Epsilon-regularity for the solutions of a free boundary system.* **Rev. Mat. Iberoam.** 39 (2023), no. 5, 1947–1972.(2023).

[A42] N. Edelen, L. Spolaor, B. Velichkov : *A strong maximum principle for minimizers of the one-phase Bernoulli problem.* **Indiana Univ. Math. J.** *73 (2024), 1061–1096 (2024)* .

[A41] F. Générau, E. Oudet, B. Velichkov : *Numerical computation of the cut locus via a variational approximation of the distance function.* **ESAIM:M2AN.** *56 (1) (2022), 105–120.*

[A40] F. Générau, E. Oudet, B. Velichkov : *Cut locus on compact manifolds and uniform semiconcavity estimates for a variational inequality.* **Arch. Rat. Mech. Anal.** *246 (2022), 561–602.*

[A39] F. Maiale, G. Tortone, B. Velichkov : *The boundary Harnack principle on optimal domains.* **Annali della Scuola Normale Superiore Cl. Sci** 25 (2024), 127–149.

[A38] G. Buttazzo, F. Maiale, B. Velichkov : *Shape optimization problems in control form.* **Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur.** 32 (2021), 413–435.

[A37] D. Mazzoleni, B. Trey, B. Velichkov : *Regularity of the optimal sets for the second Dirichlet eigenvalue.* **Ann. Inst. H. Poincaré Anal. Non Linéaire** 39 (3) (2022), 529–573.

[A36] G. De Philippis, L. Spolaor, B. Velichkov : *Regularity of the free boundary for the two-phase Bernoulli problem.* **Invent. Math.** 225 (2021), 347–394.

[A35] M. Colombo, L. Spolaor, B. Velichkov : *Almost everywhere uniqueness of blow-up limits for the lower dimensional obstacle problem.* **Interfaces and Free Bound.** 23 (2) (2021), 159–167.

[A34] S. Guarino Lo Bianco, D. A. La Manna, B. Velichkov : *A two-phase problem with Robin conditions on the free boundary.* **Journal de l'École Polytechnique** 8 (2021), 1–25.

[A33] L. Spolaor, B. Velichkov : *On the logarithmic epiperimetric inequality for the obstacle problem.* **Math. in Engineering** 3 (1) (2021), 1–42. Special issue dedicated to Sandro Salsa.

[A32] M. Engelstein, L. Spolaor, B. Velichkov : *Uniqueness of the blow-up at isolated singularities for the Alt-Caffarelli functional.* **Duke Math. J.** 169 (8) (2020), 1541–1601.

[A31] E. Russ, B. Trey, B. Velichkov : *Existence and regularity of optimal shapes for elliptic operators with drift.* **Calc. Var. PDE** 58 (2019).

[A30] M. Colombo, L. Spolaor, B. Velichkov : *On the asymptotic behavior of the solutions to parabolic variational inequalities.* **J. reine agnew. Math.** 768 (2020), 149–182.

[A29] L. Spolaor, B. Trey, B. Velichkov : *Free boundary regularity for a multiphase shape optimization problem.* **Comm. PDE** 45 (2) (2020), 77–108.

[A28] D. Mazzoleni, S. Terracini, B. Velichkov : *Regularity of the free boundary for the vectorial Bernoulli problem.* **Anal. PDE** 13 (3) (2020), 741–764.

[A27] M. Colombo, L. Spolaor, B. Velichkov : *Direct epiperimetric inequalities for the thin obstacle problem and applications.* **Comm. Pure. Appl. Math.** 73 (2) (2020), 384–420.

[A26] M. Engelstein, L. Spolaor, B. Velichkov : *(Log-)epiperimetric inequality and regularity over smooth cones for almost area-minimizing currents.* **Geometry & Topology** 23 (2019), 513–540.

[A25] I. Fragalà, B. Velichkov : *Serrin-type theorems for triangles.* **Proc. Amer. Math. Soc.** 147 (2019), 1615–1626.

[A24] G. Buttazzo, F. Maestre, B. Velichkov : *Optimal potentials for problems with changing sing data.* **J. Optim. Theory Appl.** 178 (3) (2018), 742–762.

[A23] M. Colombo, L. Spolaor, B. Velichkov : *A logarithmic epiperimetric inequality for the obstacle problem.* **Geom. Funct. Anal.** 28 (4) (2018), 1029–1061.

[A22] G. Buttazzo, B. Velichkov : *A shape optimal control problem with changing sign data.* **SIAM J. Math. Anal.** 50 (3) (2018), 2608–2627.

[A21] L. Spolaor, B. Velichkov : *An epiperimetric inequality for the regularity of some free boundary problems: the 2-dimensional case.* **Comm. Pure. Appl. Math.** 72 (2) (2018), 375–421.

[A20] G. De Philippis, J. Lamboley, M. Pierre, B. Velichkov : *Regularity of minimizers of shape optimization problems involving perimeter.* **J. Math. Pure. Appl.** 109 (2018), 147–181.

[A19] D. Bucur, I. Fragalà, B. Velichkov, G. Verzini : *On the honeycomb conjecture for a class of minimal convex partitions.* **Trans. Amer. Math. Soc.** 370 (10) (2018), 7149–7179.

[A18] D. Mazzoleni, S. Terracini, B. Velichkov : *Regularity of the optimal sets for some spectral functionals.* **Geom. Funct. Anal.** 27 (2017), 373–426.

[A17] A. Massaccesi, E. Oudet, B. Velichkov : *Numerical Calibration of Steiner trees.* **Appl. Math. Optim.** 79 (2019), 69–86.

[A16] J.C. Bellido, G. Buttazzo, B. Velichkov : *Worst-case shape optimization for the Dirichlet energy.* **Nonlinear Analysis** 153 (2017), 117–129.

[A15] G. De Philippis, A. Meszaros, F. Santambrogio, B. Velichkov : *BV Estimates in Optimal Transportation and Applications.* **Arch. Rat. Mech. Anal.** 219 (2) (2016), 829–860.

[A14] D. Bucur, D. Mazzoleni, A. Pratelli, B. Velichkov : *Lipschitz Regularity of the Eigenfunctions on Optimal Domains.* **Arch. Rat. Mech. Anal.** 216 (2015), 117–151.

[A13] L. Brasco, G. De Philippis, B. Velichkov : *Faber-Krahn inequalities in sharp quantitative form.* **Duke Math. J.** 164 (9) (2015), 1777–1831.

[A12] G. Buttazzo, E. Oudet, B. Velichkov : *A free boundary problem arising in PDE optimization.* **Calc. Var. PDE** 54 (4) (2015), 3829–3856.

[A11] B. Bogosel, B. Velichkov : *Multiphase Optimization Problems for Eigenvalues: Qualitative Properties and Numerical Results.* **SIAM J. Numer. Anal.** 54 (1) (2015), 210–241.

[A10] D. Bucur, B. Velichkov : *A free boundary approach to shape optimisation problems.* **Phil. Trans. R. Soc. A** 373 (2015).

[A9] G. De Philippis, B. Velichkov : *Existence and regularity of minimizers for some spectral optimization problems with perimeter constraint.* **Appl. Math. Optim.** 69 (2) (2014), 199–231.

[A8] G. Buttazzo, B. Ruffini, B. Velichkov : *Spectral optimization problems for metric graphs.* **ESAIM: COCV** 20 (1) (2014) 1–22.

[A7] B. Velichkov : *A Note on the Monotonicity Formula of Caffarelli-Jerison-Kenig.* **Rend. Lincei Mat. Appl.** 25 (2014), 165–189.

[A6] G. Buttazzo, A. Gerolin, B. Ruffini, B. Velichkov : *Optimal potentials for Schrödinger operators.* **Journal de l'École Polytechnique** 1 (2014), 71–100.

[A5] D. Bucur, G. Buttazzo, B. Velichkov : *Spectral Optimization Problems for Potentials and Measures.* **SIAM J. Math. Anal.** 46 (4) (2014), 2956–2986.

[A4] D. Bucur, B. Velichkov : *Multiphase shape optimization problems.* **SIAM J. Control Optim.** 52 (6) (2014), 3556–3591.

[A3] G. Buttazzo, B. Velichkov : *Some new problems in spectral optimization.* **Banach Center Publications** 101 (2014), 19–35.

[A2] D. Bucur, G. Buttazzo, B. Velichkov : *Spectral optimization problems with internal constraint.*
Ann. Inst. H. Poincaré Anal. Non Linéaire 30 (3) (2013), 477–495.

[A1] G. Buttazzo, B. Velichkov : *Shape optimization problems on metric measure spaces.*
J. Funct. Anal. 264 (1) (2013), 1–33.

Book chapters

[B4] G. Buttazzo, B. Velichkov : *Spectral optimization problems for Schrödinger operators.*
Shape Optimization and Spectral Theory, De Gruyter (2017), 325–352.

[B3] G. Buttazzo, B. Velichkov : *The spectral drop problem.*
Contemp. Math. **666** (2016), 111–135.

[B2] G. Buttazzo, M. Van Den Berg, B. Velichkov : *Optimization problems involving the first Dirichlet eigenvalue and the torsional rigidity.* New Trends in Shape Optim., Springer (2015), 19–41.

[B1] V. Georgiev, B. Velichkov : *Decay estimates for the supercritical 3-D Schrödinger equation with rapidly decreasing potential.* Progr. in Math. **301** (2012), 145–162.

Monographs

[M2] B. Velichkov : *Regularity of the one-phase free boundaries.* Lecture Notes of the Unione Matematica Italiana - UMILN (volume 28), Springer 2023, ISBN 978-3-031-13237-7.
Open access: `https://link.springer.com/book/10.1007/978-3-031-13238-4`

[M1] B. Velichkov : *Existence and regularity results for some shape optimization problems.*
Edizioni della Normale, Tesi 19, Springer 2015, ISBN 978-88-7642-526-4.

List of seminars and talks at conferences

Mini courses

Bernoulli free boundary systems (**6 hours mini-cours** - *Link.*)
School on "Selected topics in Calculus of Variations and PDEs" (Pisa, 2025).

Free boundary regularity for the one-phase Bernoulli problem (**6 hours mini-cours** - *Link.*)
Free boundary problems and related topics (ETH - Zürich, 2022).

Regularity of the one-phase free boundaries. (**6 hours mini-cours** - *Link.*)
Shape optimization, control and inverse problems for PDEs (Naples, 2019).

Talks at conferences and workshops

On the boundary branching set of the one-phase problem.
XXXV Convegno Nazionale di Calcolo delle Variazioni (Riccione, 2025).

Regularity up to the boundary for optimal partition problems.
Workshop Calculus of Variations (Oberwolfach 2024).

Regular and singular one-phase free boundaries.
Geometric Measure Theory and applications 2024 (17–21 June 2024, Cortona).

Optimal partition problems.
50 anni di Calcolo delle Variazioni - workshop in honour of Giuseppe Buttazzo (Pisa 2024).

Regular and singular one-phase free boundaries.
New Trends in Nonlinear PDE's, Physics and Geometry (Granada 2024) Link.

Regular and singular one-phase free boundaries.
Shape Optimization and Isoperimetric and Functional Inequalities (Levico 2023) Link.

On the fine structure of the two-phase free boundaries.
Workshop Partial Differentials Equations (Oberwolfach 2023) Link.

On the fine structure of the two-phase free boundaries.
Highlights in Nonlinear Analysis - workshop in honour of Susanna Terracini (Cetraro 2023) Link.

On the regularity of the optimal shapes for a class of integral functionals.
New Trends in Geometric & Variational PDEs (Trento 2023) Link.

On the regularity of the optimal shapes for a class of integral functionals.
Shape Optimisation and Geometric Spectral Theory (Edinburgh 2022) Link.

On the regularity of the optimal shapes for a class of integral functionals.
IX Partial differential equations, optimal design and numerics (Benasque 2022) Link.

Free boundary clusters with two phases.
Geometric Analysis and PDE on Garda Lake (Gargnano, 2022) Link.

Free boundary clusters with two phases.
MSRI Workshop - Hot Topics:
Regularity Theory for Minimal Surfaces and Mean Curvature Flow (online, 2022). Link.

An epsilon-regularity theorem for a free boundary system.
Geometric Measure Theory and applications (Cortona, 2021). Link.

An epsilon-regularity theorem for a free boundary system.
Workshop Partial Differentials Equations (Oberwolfach, 2021). Link.

Regularity of the two-phase free boundaries.
Workshop Calculus of Variations (Oberwolfach, 2020). Link.

Regularity of the two-phase free boundaries.
XXX Convegno Nazionale di Calcolo delle Variazioni (Levico Terme, 2020). Link.

Regularity of the two-phase free boundaries.
Workshop Geometric Measure Theory in Padova (Padova, 2020). Link.

Regularity of the two-phase free boundaries.
Shape optimization, Isoperimetric and Functional Inequalities (Levico Terme, 2019) Link.

On the logarithmic epiperimetric inequality.
Partial Differentials Equations (Oberwolfach, 2019) Link.

On the logarithmic epiperimetric inequality.
Brescia - Trento Nonlinear Day III (Trento, 2019). Link.

On the logarithmic epiperimetric inequality.
Analytic and Geometric Aspects of PDEs (Milano, 2019). Link.

On the logarithmic epiperimetric inequality.
XXIX Convegno Nazionale di Calcolo delle Variazioni (Levico Terme, 2019). Link.

Recent results on the regularity of the free boundary of the obstacle problem.
Calculus of Variations at Paris-Diderot (Paris, 2018). Link.

Shape optimization problems for elliptic operators with drift.
Journées Optimisation de Formes et Applications 2 (Pau, 2018). Link.

On the honeycomb conjecture for a class of minimal convex partitions.
Transport problems in Zürich (Zürich, 2017). Link.

An epiperimetric inequality approach to the regularity of the free boundaries.
GMT Shape Optimization and Free Boundaries (SISSA, Trieste, 2016). Link.

Lipschitz regularity for quasi-minimizers and applications to some shape optimization problems.
Calculus of Variations, Geometric Measure Theory, Optimal Transportation: from Theory to Applications (Lyon, 2016). Link.

Regularity of optimal sets for spectral functionals.
Advances in Nonlinear Problems from Material Science and Shape Optimization (Pisa, 2016). Link.

Regularity of optimal sets for spectral functionals.
Bru-To: Bruxelles-Torino PDE's Conference (Torino, 2016). Link.

Reinforcement of an elastic membrane, infinity laplacian and obstacle problems.
Spectral Theory and Shape Optimization Problems for Elliptic PDEs (Milano, 2015). Link.

Tre problemi di ottimizzazione legati al problema dell'ostacolo
Proprietà Analitico-Geometriche di Soluzioni di EDP (Napoli, 2016). Link.

Lipschitz continuity of the eigenfunctions on optimal sets.
Journées EDP Rhone-Alpes-Auvergne 2014 (Ecully, 2014). Link.

Short talks (20-30 min) at conferences and workshops

On the logarithmic epiperimetric inequality.
Conference DEA - Dynamics, Equations and Applications, (Krakow, 2019). Link.

On the logarithmic epiperimetric inequality.
12th ISAAC Congress, (Aveiro, 2019). Link.

Variational approach to the regularity of optimal sets
VII PDE, Optimal Design and Numerics (Benasque, 2017). Link.

An epiperimetric inequality approach to the regularity of the free boundaries.
Shape Optimization and Isoperimetric and Functional Inequalities (CIRM Luminy, 2016). Link.

Regularity of the optimal sets for spectral functionals.
XIII Colloque Franco-Roumain de Mathématiques Appliquées (Iaşi, 2016). Link.

A free boundary problem arising in PDE optimization.
XXVI Convegno Nazionale di Calcolo delle Variazioni (Levico Terme, 2016). Link.

A free boundary problem arising in PDE optimization.
Shape Optimization and Spectral Geometry (ICMS Edinburgh, 2015). Link.

Lipschitz continuity of the eigenfunctions on optimal sets.
Workshop Shape and topological optimization (RICAM, Linz, 2014). Link.

Shape optimization problems for spectral functionals with perimeter and volume terms.
XXVI Convegno Nazionale di Calcolo delle Variazioni (Levico Terme, 2014). Link.

Multiphase shape optimization problems.
New Trends in Shape Optimization (Erlangen, 2013). Link.

The spectral drop problem.
Partial Differential Equations, Optimal Design and Numerics (Benasque, 2013). Link.

Shape optimization problems with internal constraint.
Shape Optimization Problems and Spectral Theory (CIRM Luminy, 2012). Link.

Shape optimization problems with internal constraint.
XXVI Convegno Nazionale di Calcolo delle Variazioni (Levico Terme, 2012). Link.

Shape Optimization Problems on Metric Measure Spaces.
Geometric Analysis of Sub-Riemannian and Metric Spaces (Pisa, 2011). Link.

Seminars

On the structure of the two-phase free boundaries in dimension two.
SISSA's Analysis seminars, 10/3/2026. Link.

Regularity up to the boundary for optimal partition problems.
Università di Milano, 28/5/2024. Link.

Free boundary clusters with two phases.
Università di Firenze, 29/4/2022. Link.

Vectorial free boundary problems.
Lisbon WADE Seminar - online, 4/3/2021. Link.

Vectorial free boundary problems and regularity of the optimal sets for the eigenvalues of the Dirichlet Laplacian. One world PDE Seminar - online, 2/3/2021. Link.

Regularity of the two-phase free boundaries.
Sapienza Università di Roma, 28/11/2019.

Regularity of the two-phase free boundaries.
Università di Pisa, 06/11/2019.

Regularity of the free boundaries around isolated singularities.
Università degli studi di Napoli Federico II, 26/03/2018.

Variational approach to the regularity of the singular free boundaries.
ETH Zürich,13/03/2018.

Regularity of the free boundaries around isolated singularities.
Calcolo delle Variazioni ed Analisi Geometrica, Università di Pisa, 07/02/2018.

Regularity of the free boundaries around isolated singularities.
Université Paris Diderot, 05/02/2018.

Regularity of the free boundaries around isolated singularities.
Université Paris Sud, 26/01/2018.

Variational approach to the regularity of the free boundaries.
Université Aix-Marseille, 26/10/2017.

Regularity of the optimal sets for spectral functionals.
Max Planck Institut Leipzig, 13/05/2016.

Regularity of the optimal sets for spectral functionals.
Unuversität Zürich, 13/04/2016.

Regularity of the optimal sets for spectral functionals.
Politecnico di Milano, 20/04/2016.

Lipschitz continuity of the eigenfunctions on optimal sets.
Università degli Studi di Torino, 29/10/2014.

Régularité Lipschitz pour les fonctions propres sur ensembles optimaux.
Séminaire Parisien d'Optimisation, 6/10/2014.

Optimal domains for spectral functionals with perimeter and volume penalizations.
Séminaires Analyse Appliquée LATP Marseille, 08/04/2014.

Optimal domains for spectral functionals with perimeter and volume penalizations.
Séminaires LJK - Géométrie - Images - Calcul des Variations, Université de Grenoble, 02/04/2014.

Existence and regularity for some spectral optimization problems with perimeter constraint.
Université de Savoie, 09/11/2012.

Subsolutions of the Dirichlet energy functional. Density estimates and applications.
Università di Pavia, 08/05/2012.

Concentration-compactness principle and shape optimization problems.
Calcolo delle Variazioni ed Analisi Geometrica, Università di Pisa, 25/05/2011.

Teaching: list of courses

2024 - 2025 (111 ore, Università di Pisa).

- "Analisi Matematica II" - Ingegneria dell'Energia (61 ore, S1);
- "Equazioni Ellittiche" - CdL Matematica (42 ore, S2);
- "Spazi di Sobolev" - PhD in Mathemics (8 ore, S2).

2023 - 2024 (150 ore, Università di Pisa).

- "Analisi Matematica II" - Ingegneria dell'Energia (60 ore, S1);
- "Spazi di Sobolev" - CdL Matematica (48 ore, S2);
- "Equazioni Ellittiche" - CdL Matematica (42 ore, S2).

2022 - 2023 (120 ore, Università di Pisa).

- "Analisi Matematica II" - Ingegneria dell'Energia (60 ore, S1);
- "Spazi di Sobolev" - CdL Matematica (48 ore, S2);
- "Monotonicity formulas in free boundary and geometric variational problems" - PhD in Mathematics (12 ore, S2).

2021 - 2022 (120 ore, Università di Pisa).

- "Complementi di Analisi Matematica" (Analisi 2) - CdL Fisica (54 ore);
- "Analisi Matematica II" - Ingegneria dell'Energia (30 ore);
- "Calcolo delle Variazioni A" - CdL Matematica (42 ore).

2020 - 2021 (102 ore, Università di Pisa).

- "Analisi Matematica II" - Ingegneria dell'Energia (60 ore);
- "Calcolo delle Variazioni A" - CdL Matematica (42 ore).

2019 - 2020 (120 ore, Università degli Studi di Napoli "Federico II").

- "Analisi Matematica I" - Ingegneria (60 ore, S1);
- "Analisi Matematica II" - Ingegneria (60 ore, S2).

2018 - 2019 (96 ore, Université Grenoble Alpes).

- MAT102 Mathématiues outils pour les sciences e l'ingégnerie (96h).

2017 - 2018 (165 ore, Université Grenoble Alpes).

- MAT102 Mathématiues outils pour les sciences e l'ingégnerie (165h).

2016 - 2017 (177 ore, Université Grenoble Alpes).

- MAT102 Mathématiues outils pour les sciences e l'ingégnerie (132h);
- Introduction aux equations différentielles ordinaires et partielles (45h).

2015 - 2016 (144 ore, Université Grenoble Alpes).

- MAT102 Mathématiues outils pour les sciences e l'ingégnerie (66h);
- MAT115 Algèbre, géométrie et calcul infinitésimal pour la physique (33h);
- Introduction aux equations différentielles ordinaires et partielles (45h).

2014 - 2015 (111 ore, Université Grenoble Alpes).

- MAT115 Algèbre, géométrie et calcul infinitésimal pour la physique (66h);
- Introduction aux equations différentielles ordinaires et partielles (45h).

Teaching: supervision of undergraduate and master thesis

2024. Alberto Pacati (Università di Pisa) – Laurea Magistrale in Matematica (master thesis);
2024. Marco Lecci (Università di Pisa) – Laurea Magistrale in Matematica (master thesis);
2024. Andrea Rocca (Università di Pisa) – Laurea Triennale in Matematica (undergraduate thesis);
2023. Filippo Paiano (Università di Pisa) – Laurea Magistrale in Matematica (master thesis)
co-supervised with Joaquim Serra (ETH).