Book of abstracts

Regularity and geometric aspects of nonlinear PDEs II

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Speakers:

- Alessandro Audrito (Politecnico di Torino);
- Tommaso Bertin (Università di Padova);
- Gianmarco Caldini (Università di Trento);
- Filomena De Filippis (University of Salzburg);
- Alessandra De Luca (Università di Milano-Bicocca);
- Gabriele Fioravanti (Università di Torino);
- Lorenzo Giaretto (Università di Torino);
- Pablo Hidalgo Palencia (Universitat de Barcelona);
- Dario Mazzoleni (Università di Pavia);
- Matteo Talluri (Università di Bologna);
- Gianmaria Verzini (Politecnico di Milano).

Abstracts

Existence of weak solutions in parabolic free boundary problems

Alessandro Audrito

Abstract: I will present some recent results about existence of weak solutions to the singular parabolic free boundary problem

$$\partial_t u - \Delta u = -\frac{\mathrm{d}}{\mathrm{d}u} u_+^\gamma,$$

where $\gamma \in (0,1]$, $u_+ := \max\{u,0\}$, and the term in the right-hand side denotes the formal derivative of the *non-smooth* function $u \mapsto u_+^\gamma$. All the results have been obtained in collaboration with Tomás Sanz-Perela (UB).

Lavrentiev Phenomenon and integral representation of the lower semicontinuous envelope for functionals with non convex and non continuous Lagrangians

Tommaso Bertin

Abstract: In this seminar, we investigate classical integral functionals in the Calculus of Variations and study conditions on the Lagrangian that guarantee the absence of the Lavrentiev Phenomenon. In particular, we focus on Lagrangians that are non-convex, possibly highly discontinuous, and unbounded with respect to the gradient variable. Our first step to prove the non-occurrence of the Lavrentiev Phenomenon is to prove the representation of the lower semicontinuous envelope of a functional defined in $W^{1,\infty}(\Omega)$ as an integral functional whose Lagrangian is given by the bipolar of the original one. Specifically, we adapt a technique presented in the classical book by Ekeland and Temam for the bounded case, and a refinement of a method due to Cellina, for the unbounded case. The integral representation also allows us to apply recent results by Bousquet, Mariconda and Treu to the non-convex setting. We establish the integral representation of the lower semicontinuous envelope and the corresponding absence of the Lavrentiev Phenomenon under very weak assumptions in the autonomous case and under suitable anti-jump conditions on the spatial variable in the non-autonomous case. Furthermore we prove the strong convergence of the approximating sequence for minimizers or under specific growth assumptions on the Lagrangian. These results provide new insights into the interplay between regularity assumptions, relaxation methods, and the avoidance of the Lavrentiev Phenomenon, thereby extending recent advances to a broader class of variational problems. This is a joint project with with G. Treu (Universita di Padova) and P. Huguet (Université de Toulouse III Paul Sabatier).

On regularity theory for generalized surfaces

Gianmarco Caldini

Abstract:The natural question of how much smoother integral currents are with respect to their initial definition goes back to the origin of the theory with the seminal article of Federer and Fleming. In this seminar I will explain how closely one can approximate integral cycles by smooth submanifolds. Our result implies the absence of the Lavrentiev phenomenon in the Plateau problem for integral cycles and smooth submanifolds. Part of what will be discussed is derived from a joint study with William Browder and Camillo De Lellis, and builds on earlier preliminary work by the former author together with Frederick Almgren.

Intrinsic Schauder estimates at nearly linear growth

Filomena De Filippis

Abstract: Variational integrals at nearly linear growth appear in the theory of plasticity with logarithmic hardening, that is the borderline configuration between plasticity with power hardening and perfect plasticity. The related (very challenging) regularity theory for minima has been intensively developed over the last 25 years, see e.g. the work of Frehse & Seregin '99, Fuchs & Mingione '00, Bildhauer '03, Beck & Bulíček & Gmeineder '20, Di Marco & Marcellini '20, Gmeineder & Kristensen '22, De Filippis & Mingione '23.

In this talk, we will discuss a nonlinear potential theoretic framework for Schauder estimates for vector valued solutions of a broad class of nonautonomous variational problems at nearly linear growth. Our approach naturally embraces the variable exponent as well as the Double and Multi phase setting, yielding new regularity results in basic models and recovering optimal regularity recently established in specific cases. From recent, joint work with Cristiana De Filippis (Parma) and Peter Hästö (Helsinki).

Unique continuation from the boundary for the spectral fractional Laplacian

Alessandra De Luca

Abstract: I will show the validity of the strong unique continuation property at certain boundary points for solutions to a class of nonlocal elliptic equations governed by the spectral fractional Laplace operator.

An extension procedure leads to study an equivalent local problem in one dimension more on a cylinder, with a homogeneous Dirichlet boundary condition on the lateral surface and a non-homogeneous Neumann condition on the basis. For the extended problem, after an odd reflection through the boundary, enough regularity is available to derive a Pohozaev-type identity and consequently some doubling properties via an Almgren-type monotonicity formula.

An additional blow-up analysis allows us to classify all the admissible vanishing orders of solutions at the edge and thus to get the strong unique continuation property for the nonlocal problem as well.

Regularity for degenerate and singular parabolic equations

Gabriele Fioravanti

Abstract: In this talk we present some regularity results about the following class of parabolic PDEs

$$y^a\partial_t u - \mathrm{div}(y^a A\nabla u) = y^a f + \mathrm{div}(y^a F) \quad \text{in} \quad B_1^+ \times (-1,1), \tag{1}$$

where $y > 0$ and $a > -1$. The presence of the weight $y^a \in L^1$ affects the uniform ellipticity of the operator on the hyperplane $\Sigma := y = 0$, since y^a can vanishes or explode as $y \to 0^+$. We investigate the regularity up to boundary Σ of solutions to (1), assuming they satisfy a conformal-type boundary condition on Σ. Our main result is a complete Schauder regularity theory in the class of parabolic Hölder spaces $C_p^{k,\alpha}$.

This is a joint project with with A. Audrito (Politecnico di Torino) and S. Vita (Università di Pavia).

Least energy solutions for nonlinear Schrödinger system with K-wise interactions and related free boundary problems

Lorenzo Giaretto

Abstract: In the first part of this talk we discuss the existence and qualitative properties of least energy solutions for a weakly coupled nonlinear Schrödinger system with K-wise interaction, namely systems whose interaction term involves the product of all components. We consider both attractive and repulsive regimes and provide sufficient conditions on the competition parameter ensuring the existence of least energy fully non-trivial solutions, possibly under a radial symmetry constraint.

We then investigate the asymptotic behavior of least energy fully non-trivial radial solutions in the strong competition limit. In this regime, we observe partial segregation phenomena which differ substantially from those arising in systems with pairwise interactions. In the final part of the talk, we consider a related system and establish Hölder bounds for least energy solutions that are uniform with respect to the competition parameter. We also discuss ongoing work concerning the regularity of the limiting problem and the associated free boundary.

This talk is based on joint work with N. Soave.

Positive solutions to the semilinear one-phase problem

Pablo Hidalgo-Palencia

Abstract: In his seminal 1971 work, Serrin proved that overdetermined problems are inherently rigid: under certain conditions, the only possible solutions are the trivial ones. But if we consider more general families of equations, does this rigidity still hold? More fundamentally, do solutions even exist?

In this talk, we show that a broad class of semilinear one-phase problems indeed admit solutions, and that these satisfy the classical regularity properties first explored by Alt and Caffarelli ('81). A key novelty of the work is that we do not restrict ourselves to an ambient domain; instead, we work on the whole space without boundary conditions, which requires new compactness arguments.

This is joint work with Alberto Enciso and Xavier Ros-Oton.

Existence (and nonexistence)
for functionals with competing attractive and repulsive interactions

Dario Mazzoleni

Abstract: In the last few years the Gamow problem, namely

$$\min\Big\{P(\Omega)+\varepsilon\int_\Omega\int_\Omega\frac{1}{|x-y|}\,dx\,dy:\Omega\subset\mathbb{R}^3,|\Omega|=1\Big\},$$

for $\varepsilon>0$, has attracted a lot of attention from mathematicians. Nowadays it is well understood that for small ε there exist a minimizer and it is a ball, while for very large ε there is no minimizer.

Although it is very easy to formulate, there are still several open problems about it (mostly concerning nonexistence of minimizers for large ε in a generalized N-dimensional setting with α Riesz energy).

Moreover, we will consider a different problem but with a similar structure, which can be seen as the minimization of a *Hartree energy* settled in a box, namely

$$\min\Big\{ \min_{u\in H^1_0(\Omega),\ \int u^2=1} \Big\{ \int_\Omega |\nabla u|^2 + q\int_\Omega\int_\Omega \frac{u^p(x)u^p(y)}{|x-y|}\,dxdy\Big\} : \Omega\subset\mathbb{R}^3,\ |\Omega|=1\Big\},$$

for $q>0$ and $p=1$ or $p=2$. The study of this functional, in the case $p=2$, arises when describing the ground state of a superconducting charge qubit.

In both cases ($p=1$, which requires some more work due to the possibility of sing-changing functions, and $p=2$) we show that there is a threshold $q_1>0$ such that for all $q\le q_1$ existence of minimizers occurs and minimizers are $C^{2,\gamma}$ nearly spherical.

If time permits, will also give some ideas (although nonconclusive) on how to treat the nonexistence issue for this functional.

The techniques and tools needed in the proofs are very broad. We employ spectral quantitative inequalities, the regularity of free boundaries, spectral surgery arguments and shape variations.

This talk is based on joint works with Riccardo Moraschi (Pavia), Cyrill Muratov (Pisa), Aldo Pratelli (Pisa) and Berardo Ruffini (Bologna).

Fractional Poincaré Constants and Capacitary Notions of Inradius

Matteo Talluri

Abstract: In this talk, we present estimates for the sharp constants in fractional Poincaré-Sobolev inequalities associated with an open set, in terms of a nonlocal capacitary extension of the inradius. Our approach is based on a new Maz'ya-Poincaré inequality and, as a further result, we also obtain new fractional Poincaré-Wirtinger-type estimates on balls. These inequalities display sharp limiting behaviors with respect to the fractional order of differentiability. Finally, we derive a new criterion for the embedding of the homogeneous Sobolev space $\mathcal{D}^{s,p}_0(\Omega)$ into $L^q(\Omega)$, valid in the subcritical regime, as well as a characterization of the positivity of the fractional Cheeger constant.
Based on joint project with Francesco Bozzola.

A singular perturbation approach
to some shape optimization problems arising in population dynamic

Gianmaria Verzini

Abstract: When analyzing persistence/extinction of a species in population dynamics, one is led to consider the principal eigenvalue of some indefinite weighted problems in a bounded domain. The minimization of such eigenvalue, to foster population persistence, translates into a shape optimization problem involving the subregion of the habitat which is favorable to the species.

We perform the analysis of the singular limit of this problem, associated with either Dirichlet or Neumann boundary conditions, in case of arbitrarily small favorable region. We show that, in this regime, the favorable region is connected, and it concentrates at points depending on the boundary conditions. Moreover, we investigate the interplay between the location of the favorable region and its shape. Joint works with Lorenzo Ferreri, Dario Mazzoleni and Benedetta Pellacci.
