Book of abstracts and timetable

Workshop

PDE Days in Maremma

(Maremma Matematica)

28 – 30 October 2024, Terme di Sorano

TIMETABLE

Monday 28/10/2024

09:00–10:00 Berti
10:00–11:00 Wiedemann
11:00–12:00 Daneri
16:00–17:00 Ahmedou
17:00–18:00 Lenzmann

Tuesday 29/10/2024

09:00–10:00 Bucur
10:00–11:00 Cabré
11:00–12:00 Verzini
16:00–17:00 Burq
17:00–18:00 D'Ancona

Wednesday 30/10/2024

09:00–10:00 Rota Nodari
10:00–11:00 Terracini

This workshop is organized with the support of the projects:

- *Nonlinear dispersive equations and dynamics of fluids.*
  Progetti di Ricerca di Ateneo PRA 2022 (Università di Pisa, PI Bellazzini)
- *Geometric evolution problems and PDEs on variable domains.*
  Progetti di Ricerca di Ateneo PRA 2022 (Università di Pisa, PI Velichkov)
- ERC *"VAREG – Variational approach to the regularity of the free boundaries"* (n.853404), granted by the European Research Council (ERC) within the European Union's Horizon 2020 research and innovation programme.

Organizers: Jacopo Bellazzini, Marco Ghimenti, Bozhidar Velichkov

SPEAKERS

PDE Days in Maremma

(Maremma Matematica)

28 – 30 October 2024

Mohameden Ahmedou (Giessen University)

Massimiliano Berti (SISSA)

Dorin Bucur (Université de Savoie)

Nicolas Burq (Université Paris-Sud)

Xavier Cabré (ICREA & Universitat Politècnica de Catalunya. Barcelona)

Piero D'Ancona (Università di Roma La Sapienza)

Sara Daneri (GSSI)

Enno Lenzmann (Universität Basel)

Simona Rota Nodari (Université Côte d'Azur)

Susanna Terracini (Università di Torino)

Gianmaria Verzini (Politecnico di Milano)

Emil Wiedemann (Friedrich-Alexander-Universität Erlangen-Nürnberg)

PARTICIPANTS

PDE Days in Maremma

(Maremma Matematica)

28 – 30 October 2024

Mohameden AHMEDOU (Giessen)
Jacopo BELLAZZINI (Pisa)
Luca BENATTI (Pisa)
Massimiliano BERTI (Trieste)
Giulia BEVILACQUA (Pisa)
Dorin BUCUR (Chambery)
Nicolas BURQ (Paris)
Xavier CABRE (Barcelona)
Matteo CARDUCCI (Pisa)
Elisabetta CHIODAROLI (Pisa)
Piero D'ANCONA (Roma)
Sara DANERI (L'Aquila)
Lorenzo FERRERI (Pisa)
Luigi FORCELLA (Pisa)
Carlo GASPARETTO (Pisa)
Marco GHIMENTI (Pisa)
Marina GHISI (Pisa)
Massimo GOBBINO (Pisa)
Michele GORINI (Pisa)
Carlo GRISANTI (Pisa)
Enno LENZMANN (Basel)
Ilaria LUCARDESI (Pisa)
Roberto OGNIBENE (Pisa)
Filippo PAIANO (Pisa)
Emanuele PAOLINI (Pisa)
Alessandra PLUDA (Pisa)
Francesca PRINARI (Pisa)
Andrea ROCCA (Pisa)
Simona ROTA NODARI (Nizza)
Susanna TERRACINI (Torino)
Giorgio TORTONE (Pisa)
Bozhidar VELICHKOV (Pisa)
Gianmaria VERZINI (Milano)
Nicola VISCIGLIA (Pisa)
Emil WIEDEMANN (Erlangen)

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ABSTRACTS

PDE Days in Maremma (Maremma Matematica)

28 – 30 Oct 2024

Critical point theory at infinity: an abstract framework and applications

Mohameden Ahmedou

Abstract: In this talk, we introduce a new abstract framework that allows the application of Morse theory to certain non-compact variational problems. This is achieved by analyzing the topological changes induced by the so-called 'critical points at infinity.' As an application, we show how this approach can be used to study the Nirenberg problem on spheres and the resonant prescribed Q-curvature problem on four-dimensional manifolds.

This is a joint work with Thomas Bartsch (University of Giessen).

Quasi-periodic vortex patches of 2d Euler

Massimiliano Berti

Abstract: In this seminar I will discuss the existence of time quasi-periodic vortex patch solutions of the 2d-Euler equations in $\mathbb{R}^2$, close to uniformly rotating Kirchhoff elliptical vortices, with aspect ratios belonging to a set of asymptotically full Lebesgue measure. A major difficulty of the KAM proof is the presence of a zero normal mode frequency, which is due to the conservation of the angular momentum. The key novelty to overcome this degeneracy is to perform a perturbative symplectic reduction of the angular momentum, introducing it as a symplectic variable in the spirit of the Darboux-Carathéodory theorem of symplectic rectification, valid in finite dimension. This approach is particularly delicate in a infinite dimensional phase space: our symplectic change of variables is a nonlinear modification of the transport flow generated by the angular momentum itself. This is the first time such an idea is implemented in KAM for PDEs.

The geometric size of the fundamental gap

Dorin Bucur

Abstract: The fundamental gap conjecture proved by Andrews and Clutterbuck in 2011 provides the sharp lower bound for the difference between the first two Dirichlet Laplacian eigenvalues in terms of the diameter of a convex set in $\mathbb{R}^N$. The question concerning the rigidity of the inequality, raised by Yau in 1990, was left open. Going beyond rigidity, our main result strengthens Andrews-Clutterbuck inequality, by quantifying geometrically the excess of the gap compared to the diameter in terms of flatness. The proof relies on a localized, variational interpretation of the fundamental gap, allowing a dimension reduction via the use of convex partitions à la Payne-Weinberger: the result stems by combining a new sharp result for one dimensional Schrödinger eigenvalues with measure potentials, with a thorough analysis of the geometry of the partition into convex cells. As a by-product of our approach, we obtain a quantitative form of Payne-Weinberger inequality for the first nontrivial Neumann eigenvalue of a convex set in $\mathbb{R}^N$, thus proving, in a stronger version, a conjecture from 2007 by Hang-Wang. This is a joint work with V. Amato and I. Fragala.

Nonlinear interpolation and the flow map of quasilinear equations

Nicolas Burq

Abstract: I will present an abstract result showing that for the flow map of a quasilinear problem, both the continuity of the flow *as a function of time and the continuity of the data-to-solution map* follow automatically from the estimates that are usually proven when establishing the existence of solutions: propagation of regularity via tame a priori estimates for higher regularities and contraction for weaker norms. This result is actually the consequence of an interpolation theorem for nonlinear functionals defined on scales of Banach spaces that generalize Besov spaces.

Our analysis is self-contained and independent of any previous results about interpolation theory. It depends solely on the concepts of Friedrichs' mollifiers, seen through the formalism introduced by Hamilton, combined with the frequency envelopes introduced by Tao and used recently by two of the authors and others to study the Cauchy problem for various quasilinear evolutions in partial differential equations. Though I will not present the abstract general result, I will explain its proof and illustrate on some examples how we can easily check the assumptions.

This is a joint work with T. Alazard, M. Ifrim, D. Tataru and C. Zuily.

Stable solutions to semilinear elliptic equations are smooth up to dimension 9

Xavier Cabré

Abstract: The regularity of stable solutions to semilinear elliptic PDEs has been studied since the 1970's. It was initiated by a work of Crandall and Rabinowitz, motivated by the Gelfand problem in combustion theory. The theory experienced a revival in the mid-nineties after new progress made by Brezis and collaborators. I will present these developments, as well as a recent work, in collaboration with Figalli, Ros-Oton, and Serra, which finally establishes the regularity of stable solutions up to the optimal dimension 9. I will also describe a more recent paper of mine which provides full quantitative proofs of the regularity results.

Dispersion estimates for Dirac equations with Aharonov–Bohm magnetic fields

Piero D'Ancona

Abstract: This talk is dedicated to the dispersive properties of a two dimensional massless Dirac equation perturbed by an Aharonov–Bohm magnetic field, which is critical with respect to the scaling. Our main results will be a family of pointwise decay estimates and a full range family Strichartz estimates for the flow. These results represent the natural continuation of earlier research on evolution equations associated to operators with magnetic fields with strong singularities. The proof relies on the use of a relativistic Hankel transform, which allows for an explicit representation of the propagator in terms of the generalized eigenfunctions of the operator. This is a joint work with Federico Cacciafesta, Zhiqing Yin, and Junyong Zhang.

Pattern formation for isotropic functionals with competing interactions

Sara Daneri

Abstract: At the base of spontaneous pattern formation is universally believed to be the competition between short range attractive and long range repulsive forces. Though such a phenomenon is observed in experiments and simulations, a rigorous understanding of the mechanisms at its base is still in most physical problems a challenging open problem. The main difficulties are due to the nonlocality of the interactions and, in more than one space dimensions, the symmetry breaking phenomenon (namely the fact that the interactions have a larger group of symmetries than that of their minimizers). In this talk we consider a general class of isotropic functionals in dimension $d \geq 2$, typical in physical models, in which a surface term favouring pure phases competes with a nonlocal term with power law kernel favouring alternation between different phases. Close to the critical regime in which the two terms are of the same order, we give a rigorous proof of the conjectured symmetry breaking and pattern formation for global minimizers, in the shape of domains with flat boundary (e.g. stripes or lamellae). Among others, our approach relies on detecting a nonlocal curvature-type quantity which is controlled by the energy functional and whose finiteness implies flatness for sufficiently regular boundaries. This is a joint work with E. Runa.

Symmetry breaking for ground states of biharmonic NLS

Enno Lenzmann

Abstract: In this talk, I will discuss recently developed Fourier methods to prove and disprove symmetry results for optimisers of variational problems occurring for NLS-type equations and Gagliardo-Nirenberg interpolation inequalities. In particular, we will prove that ground state optimisers for certain biharmonic (fourth-order) NLS fail to be radially symmetric. Our proof is based on a connection to the (adjoint) Stein-Tomas inequality on the unit sphere and Knapp's counterexample. This is joint work with Tobias Weth (Frankfurt).

On a quasilinear Schrödinger equation: the small frequency limit

Simona Rota Nodari

Abstract: In this talk, I will present some recent results on a quasilinear Schrödinger equation with a power nonlinearity. After showing the uniqueness and the non-degeneracy of the positive radial solution u_ω for all $\omega > 0$, I will describe its asymptotic behavior in the limit $\omega \to 0$. This gives some important information about the orbital stability of u_ω and the uniqueness of normalized ground states. Joint work with François Genoud.

A priori regularity estimates for equations degenerating on nodal sets

Susanna Terracini

Abstract: We prove a priori and a posteriori Hölder bounds and Schauder $C^{1,\alpha}$ estimates for continuous solutions to singular/degenerate equations with variable coefficients of type

$$\operatorname{div}(|u|^a A\nabla w) = 0 \quad \text{in} \quad \Omega \subset \mathbb{R}^n,$$

where the weight u solves an elliptic equation of type $\operatorname{div}(A\nabla u) = 0$ with a Lipschitz-continuous and uniformly elliptic matrix A and has a nontrivial, possibly singular, nodal set.

Such estimates are uniform with respect to u in a class of normalized solutions having bounded Almgren's frequency. More precisely, we provide a priori Hölder bounds in any space dimension, and Schauder estimates when $n = 2$. When $a = 2$, the results apply to the ratios of two solutions to the same PDE sharing their zero sets. Then, one can infer higher order boundary Harnack principles on nodal domains by applying the Schauder estimates for solutions to the auxiliary degenerate equation. The results are based upon a fine blow-up argument, Liouville theorems and quasiconformal maps.

References:

[1] S. Terracini, G. Tortone, S. Vita. Higher order boundary Harnack principle on nodal domains via degenerate equations, to appear on Arch. Rat. Mech. Anal.

[2] S. Terracini, G. Tortone, S. Vita. A priori regularity estimates for equations degenerating on nodal sets, preprint 2024.

Singular analysis of the optimizers of the principal eigenvalue of the Laplacian with an indefinite weight

Gianmaria Verzini

Abstract: When analyzing the survival threshold for a species in population dynamics, one is led to consider the principal eigenvalue of certain indefinite weighted problems in a bounded domain. We study the minimization of such eigenvalues, associated with either Dirichlet or Neumann boundary conditions, and perform an analysis of the singular limit in the case of an arbitrarily small favorable region. We show that, in this regime, the favorable region is connected and concentrates at points depending on the boundary conditions. Moreover, we investigate the interplay between the location of the favorable region and its shape. This talk is based on joint papers with Lorenzo Ferreri, Dario Mazzoleni, and Benedetta Pellacci.

Non-Deterministic Solution Concepts in Fluid Dynamics

Emil Wiedemann

Abstract: As more and more ill-posedness results have been shown for fluid PDEs (not only by convex integration!), the idea to solve the Cauchy problem by some unique weak or entropy solution has become questionable. Instead, non-deterministic solution concepts such as measure-valued or statistical have sparked much recent research interest. They also seem to be more in line with well-known theories of turbulence, which are typically statistical. I will give an overview of such generalized solution concepts, including their weak-strong stability, their relation to more conventional solutions, and questions of existence.
