Book of abstracts

Regularity theory for free boundary and geometric variational problems VI

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Allen–Cahn solutions from the Plateau problem

Marco Badran

The Allen–Cahn model for phase transitions is one of the most extensively studied PDEs in geometric analysis, largely because its solutions can approximate minimal hypersurfaces through their level sets. This perspective has led to remarkable results in the closed setting. However, the classical framework is not well suited to boundary value problems such as the Plateau problem, since a non–separating hypersurface cannot, in general, arise as the regular level set of a real–valued function. Instead, we study an Allen–Cahn equation for sections of a suitable line bundle, which removes this obstruction but introduces new geometric and analytical challenges to the classical Allen–Cahn framework.

In this talk, I will describe the model and explain how, by means of a gluing procedure, one can construct solutions to the Allen–Cahn equation that approximate any prescribed solution of the Plateau problem. This is based on joint work with Manuel del Pino and Marco Guaraco.

Uniqueness of the blow-up for some Alt-Phillips cones

Matteo Carducci

One of the central questions in geometric analysis and free boundary problems is the uniqueness of blow-ups. In this talk, we discuss some results in this direction for the Alt-Phillips functional, which leads to a free boundary problem interpolating between the obstacle problem, the Alt-Caffarelli problem and minimal surfaces. These results are obtained in a joint work with G. Tortone.

Fractional parabolic theory as a high-dimensional limit of fractional elliptic theory

Blair Davey

Experts have long realized the parallels between elliptic and parabolic theory of partial differential equations. It is well-known that elliptic theory may be considered a static, or steady-state, version of parabolic theory. And in particular, if a parabolic estimate holds, then by eliminating the time parameter, one immediately arrives at the underlying elliptic statement. Producing a parabolic statement from an elliptic statement is not as straightforward. In this talk, we discuss how a high-dimensional limiting technique can be used to prove theorems about solutions to the fractional heat equation (or its Caffarelli-Silvestre extension problem) from their elliptic analogues. This talk covers joint work with Mariana Smit Vega Garcia.

The pinning effect of dilute defects

William Feldman

We consider the Bernoulli free boundary problem with "defects", inhomogeneities in the coefficients of compact support. When the defects are small and arrayed periodically there exist plane-like solutions with a range of large-scale slopes slightly different from the background field value (this is "pinning". By studying the capacity-like effect of a single defect in the Bernoulli free boundary problem we can compute the asymptotic expansion of the interval of pinned slopes as the defect size goes to zero (at least for rational normals). This is based on joint work with Inwon Kim.

Regularity theory for capillary surfaces

Benjy Firester

In this talk, I will discuss the relationship between minimal surfaces and the Bernoulli one-phase free boundary problem through capillarity and their shared regularity theory framework. We construct non-flat minimal capillary cones with any contact angle, notably including minimizing capillary and one-phase examples with bi-orthgonal symmetry. These cones interpolate between rescalings of a singular solution to the one-phase problem and the free-boundary minimal surfaces. These methods produce new minimal and CMC surfaces in the sphere and one-phase cones with intricate topologies. We further show regularity results proving certain singular capillary cones are minimizing in ambient dimension 8 or higher, demonstrating that the regularity theory for minimizing capillary hypersurfaces can have singularities in codimension 7 and proving the optimal regularity for contact angles near $\frac{\pi}{2}$ as well as providing new singular minimizing solutions to the one-phase problem. This is joint work with R. Tsiamis and Y. Wang.

Regularity for stable phase transitions

Enric Florit-Simon

We will discuss recent progress in the regularity for stable solutions to the Allen-Cahn equation, with a focus on the connections with minimal surface theory and two influential conjectures of De Giorgi and Yau. Part of this talk is based on recent joint work with Joaquim Serra which classifies all stable solutions to Allen-Cahn with bounded energy density in four dimensions.

Nonunique tangent maps at isolated singularities of minimizing p-harmonic maps

Jonas Hirsch

This is a tribute to Brian White's genius.

The analysis of "tangent maps" at singular points of energy–minimizing maps plays an important part in our understanding of the fine structure of the singular set.

In this talk, I will present an n-dimensional manifold N such that for every admissible tuple $p < m \leq n + 1$, there is a map from B_1^m into N that is minimizing the p-energy. This map has an isolated singularity at the origin and a continuum of distinct tangent maps. The argument behind this construction is inspired by the pioneering work of B. White.

Non-minimizing and min-max solutions to Bernoulli problems

Dennis Kriventsov

Bernoulli type free boundary problems have a well-developed existence and regularity theory. Much of this, however, is restricted to the case of minimizers of the natural energy (the Alt-Caffarelli functional). I will describe a compactness and regularity theorem that applies to any critical point instead, based on a nonlinear frequency formula and Naber-Valtorta estimates. Then I will explain, via an example involving gravity water waves, how to use this theorem to find min-max type (mountain pass) solutions. This is based on joint work with Georg Weiss.

Area minimising hypersurfaces mod p do not admit immersed branch points

Paul Minter

A well-known fact in geometric analysis is that area minimising hypersurfaces are smoothly embedded outside a singular set of codimension at least 7. A key step in this is the analysis of potential 'branch points' or 'flat singular points' in the hypersurface, which are ultimately shown not to exist.

In this talk I will discuss joint work with Sidney Stanbury (University of Cambridge) in which we establish some new analogues of these results for area minimising hypersurfaces mod p. More precisely, I will discuss why area minimising hypersurfaces mod p do not admit immersed branch points, and why if they are smoothly immersed outside a closed set of vanishing (n-1)-dimensional Hausdorff measure, then they must be smoothly immersed outside a closed set of codimension at least 3. In fact, these results only rely on the stationarity, stability, and structural properties of the hypersurface, rather than specifically the minimising mod p property. I will end with discussing several remaining open questions.

Regularity for a higher order Alt-Caffarelli problem

Mickael Nahon

Consider the following functional

$$u \in H^2(D) \mapsto \int_D (|\nabla^2 u|^2 + \chi_{u\neq 0})$$

where $D \subset R^2$. This is a higher-order analogue of the Alt-Caffarelli problem, which is linked to the minimization of the drag of an obstacle in a Stokes fluid, to the optimization of the compliance of a cantilever, and to the minimization of the first buckling eigenvalue among planar set, all under area constraints.

I will give a description of the free boundary, which is expected to be a union of regular curves joined with an angle of 1.43π.

Using the methods developed for this problem, I will also explain some preliminary regularity results for general Bernoulli-type problems, including non-minimizing solutions.

This talk combines joint projects with Jimmy Lamboley, Guido De Philippis and Jonas Hirsch.

On the energy structure of the Stefan problem

Filippo Paiano

In this talk, I will introduce the enthalpy formulation of the Stefan problem and discuss its energy dissipation law for its *total enthalpy.* I then exploit this to obtain quantitative estimates on the asymptotic behavior of the free boundaries.

This talk is based on a joint work with Bozhidar Velichkov (University of Pisa).

Generic regularity of isoperimetric regions in dimension 8

Davide Parise

Isoperimetric regions arise as minimisers of boundary area for a fixed enclosed volume, with sharp regularity theory, as established by Gonzalez, Massari, and Tamanini. In particular, the boundary of such a region is a smooth hypersurface away from a closed singular set of codimension seven. In closed Riemannan manifolds of dimension eight, this singular set consists of at most finitely many isolated points, with explicit singular examples having been constructed recently. In this talk, I will explain how under some assumptions on the choice of the ambient metric and enclosed volume, these singularities can be perturbed away, thus implying that the boundary is (generically) a smooth hypersurface. This is based on joint work with Kobe Marshall-Stevens and Gongping Niu.

Rigidity theorems for critical points of capillary energies

Reinaldo Resende

We will introduce the framework of capillary energies in the class of sets of finite perimeter inside an open half-space. These energies take into account both the free surface energy, namely the perimeter inside the half-space, and the wetting energy, namely the perimeter on the boundary of the half-space. The presence of the wetting region naturally leads to a free-boundary problem. We will then discuss an Alexandrov-type theorem that characterizes all critical points of these energies among sets of finite perimeter, showing that they must be suitable unions of balls and caps of balls. Afterward, we will discuss an anisotropic version of this theorem, which characterizes all critical points for the elliptic version of these capillary energies so called anisotropic capillary energies. This is joint work with A. De Rosa and R. Neumayer.

Optimal regularity for the variable coefficients parabolic Signorini problem

Wenhui Shi

In this talk I will discuss the optimal regularity of solutions to the variable coefficient parabolic Signorini problem, where the coefficients are in the Sobolev space $W_p^{1,1}$ with $p > n + 2$ (here n is the space dimension). I will talk about two main ingredients used in the proof: a parabolic Carleman estimate, and an energy/energy-dissipation inequality. This is based on joint work with Vedansh Arya.

A quantitative Alexandrov inequality with applications to geometric flows in 3D

Emanuele Spadaro

I will present a sharp quantitative version of the Alexandrov theorem on closed hypersurfaces with constant mean curvature, with applications to the analysis of the asymptotic behavior of the volume preserving and the Mullins-Sekerka flat flows.

Ilmanen's elliptic regularization scheme for anisotropic mean curvature flows

Salvatore Stuvard

In his pioneering 1994 work, Tom Ilmanen introduced a robust method, based on elliptic regularization, for constructing solutions to the isotropic mean curvature flow of possibly singular cycles in Euclidean spaces and Riemannian manifolds. Since then, the method has served as a gold standard for producing Brakke flows with good continuity properties in time.

A full anisotropic counterpart, however, faces substantial difficulties. These difficulties arise from the lack of structural tools available for the area integrand, and persist even when the anisotropy is smooth away from the origin. In this talk, I will explain how to overcome these obstacles in the codimension-one setting of boundaries of sets of finite perimeter whose level-set generalized motion does not fatten.

More precisely, using an approach that is new even in the isotropic case, I will prove that, under a non-fattening assumption on the initial datum, the anisotropic elliptic regularization scheme converges in the Kuratowski sense to the corresponding non-fattening level-set solution. This result, combined with a new conditional compactness theorem for anisotropic almost-Brakke flows, is then used to show that every such generalized motion canonically determines a unit-density anisotropic Brakke flow, which also solves the anisotropic mean curvature flow in the BV sense.

This is joint work with C. A. Antonini, A. De Rosa, and M. Morini.

TBA

Susanna Terracini

TBA

Mean curvature flows with prescribed singular sets

Raphael Tsiamis

For every closed set $K \subset \mathbb{R}^n$ and every $m \geq 2$, we construct a mean-convex ancient mean curvature flow of hypersurfaces in $\mathbb{R}^{m+n}$, with respect to a smooth Riemannian metric arbitrarily C^∞-close to the Euclidean metric, whose first-time singular set is exactly $K \times \{0\}$.

A boundary monotonicity formula for almost-minimizers of the relative perimeter in nonsmooth domains

Giacomo Vianello

Monotonicity formulas for the renormalized perimeter in balls $B(x, r)$ play a central role in the regularity theory for minimizers and almost-minimizers of the relative perimeter in an open set $\Omega \subset \mathbb{R}^n$. For interior balls $B(x, r) \subset \Omega$, such formulas are classical. However, boundary monotonicity formulas typically rely on smoothness assumptions on the container. In this talk, I will present a recent boundary monotonicity result that holds under a geometric visibility condition on Ω with respect to a point $x \in \partial\Omega$. This condition is satisfied by a large class of nonsmooth Lipschitz domains. In the final part of the presentation, I will discuss how this formula can be applied to show that, up to subsequences, rescalings of the almost-minimizer converge to a minimizing cone contained in the tangent cone to Ω at x. This is a joint collaboration with G.P. Leonardi.

Local regularity for anisotropic magnetic operators with general codimension singularities

Stefano Vita

We discuss some regularity results for a class of anisotropic magnetic Schrödinger equations, involving singular magnetic potentials. We also discuss some physical motivations coming from solenoidal Aharonov–Bohm-type models. This is a joint project with Giovanni Siclari.

Generic regularity of the free boundary in the Alt-Caffarelli-Phillips problem

Hui Yu

The Alt-Caffarelli problem and the Alt-Phillips problem are among the most well-studied elliptic free boundary problems. Much effort has been devoted to estimating the size of the singular set on the free boundary, for instance, in terms of its Hausdorff dimension.

In this talk, we discuss how such estimates can be improved for generic boundary data. This talk is based on a recent joint work with Xavier Fernández-Real at EPFL.