BOOK OF ABSTRACTS

CALCULUS OF VARIATIONS AND FREE BOUNDARY PROBLEMS:

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Organizers:

- Filippo Paiano (Università di Pisa);
- Giulia Bevilacqua (Università di Pisa);
- Bozhidar Velichkov (Università di Pisa).

Speakers:

- Giulia Bevilacqua (Università di Pisa);
- Matteo Carducci (Scuola Normale Superiore);
- Lorenzo Ferreri (Scuola Normale Superiore);
- Federico Lai (Università di Pisa);
- Roberto Ognibene (Università di Milano-Bicocca);
- Filippo Paiano (Università di Pisa);
- Maria Antonietta Palladino (Scuola Normale Superiore);
- Giovanni Siclari (Scuola Normale Superiore);
- Giorgio Tortone (Università di Torino).

Abstracts

Singularities in a free boundary problem

Giorgio Tortone

Abstract: In this short talk, we present several remarks concerning the existence of minimizing singular cones arising in the Alt-Caffarelli free boundary problem. Motivated by the classical theory of area-minimizing hypersurfaces, we first review the framework and main ideas introduced by Jerison and Savin in their study of the Alt-Caffarelli functional. We then discuss possible extensions of these ideas to more general settings. If time allows, we will also address analogous questions in related free boundary problems, such as the Alt–Phillips and capillarity problems. The "remarks" are based on ongoing joint projects with B. Velichkov (UniPi).

On the blow-up of the vectorial Bernoulli free boundary problem

Giovanni Siclari

Abstract: In this talk, we discuss the classification of the blow-up limits of minimizers of the vectorial Bernoulli free boundary problem in a bounded box D at branching points. It is known that the blow-up limits are linear and global minimizers of the vectorial Bernoulli functional

$$J_\Lambda(W, B_R) := \int_{B_R} |\nabla W|^2 \, dx + \Lambda |\Omega_W \cap B_R|,$$

for some $\Lambda > 0$. We compute, for any linear map Ax, the optimal constant

$$\Lambda^*(A) := \sup\{\Lambda > 0 : Ax \text{ is a global minimizer of } J_\Lambda\}$$

in terms of A and shows that it depends only on the branching point x_0 and not on the particular blow-up limit $A \in \mathcal{BU}_U(x_0)$. Furthermore, if time permits it, we discuss the regularity of the measure constraint vectorial Bernoulli free boundary problem and the asymptotic of minimizers as the measure constraint m tends to $|D|$. Such a study with a linear datum on the fixed boundary is the main ingredient to obtain the aforementioned results about blow-up limits.

Free boundary regularity in the thin obstacle problem

Matteo Carducci

Abstract: In this talk, we discuss the thin obstacle free boundary problem, which consists in minimizing the Dirichlet energy among functions that stay above a fixed obstacle defined on a lower-dimensional set. We provide an overview of the principal known results and describe some recent progress concerning the uniqueness of blow-ups and the structure of the free boundary.

Perimeter minimization with prescribed boundary data and volume constraint

Maria Antonietta Palladino

Abstract: We consider a perimeter-minimization problem with a fixed volume constraint, where part of the reduced boundary is prescribed and must lie on a plane. We aim to understand how this boundary constraint interacts with the volume condition and how it may influence the geometry of possible minimizers.

On generalizations of the Plateau problem

Giulia Bevilacqua

Abstract: We present two recent results extending the classical Plateau problem. First, we address the regularity of soap films with capillarity effects. In this context, soap films are modeled as sets of finite perimeter subject to a volume constraint and a topological spanning condition. Second, we characterize minimizers of an energy functional that couples the area functional with a term arising from the theory of liquid crystals.

The structure of planar free boundaries

Lorenzo Ferreri

Abstract: We study the branch point singularities of the free boundaries in the plane. We show that they are locally finite, and this leads to a complete description of their structure. We also disprove the analytic separation condition: there is no analytic curve passing through the free boundaries (no exact center manifold).

The enthalpy dissipation law for the one-phase Stefan problem.

Filippo Paiano

Abstract: The Stefan problem can be formulated in terms of the dissipation of its internal energy (enthalpy). In this talk, I will present a monotonicity formula for the total enthalpy of one-phase solutions, together with some applications to the two-phase problem with a non-empty mushy region.

Parabolic segregation problem: asymptotic aspects

Federico Lai

Abstract: We study the asymptotic behavior, with a decay rate, of the parabolic segregation problem's solution, and its elliptic regularized functions introduced by A. Audrito, E. Serra and P. Tilli in their work "A minimization procedure to the existence of segregated solutions to parabolic reaction-diffusion systems".

Open questions in the optimal partition problem

Roberto Ognibene

Abstract: We consider the problem of minimizing the Dirichlet energy of a vector-valued function whose components have disjoint support, known as the optimal partition problem. I will survey the state of the art regarding the regularity of the associated free boundaries and I will present some of the main open questions in this area.