ABSTRACTS

Calculus of Variations and Free Boundary Problems IX

20 – 21 Nov 2024

A non-parametric Plateau problem with partial free boundary

Roberta Marziani (UNISI)

We consider a Plateau problem in codimension 1 in the non-parametric setting, where a Dirichlet boundary datum is assigned only on part of the boundary $\partial\Omega$ of a bounded convex domain $\Omega \subset \mathbb{R}^2$. Where the Dirichlet datum is not prescribed, we allow a free contact with the horizontal plane. We first show existence of a solution, and prove regularity for the corresponding area-minimizing surface. Then we complement the analysis by comparing these solutions with the classical minimal surfaces of Meeks and Yau, and show that they are equivalent when the Dirichlet boundary datum is assigned on at most 2 disjoint arcs of $\partial\Omega$. A relevant consequence of this equivalence is that if the boundary datum is symmetric with respect to the horizontal plane and its upper part is cartesian, then the same holds for the corresponding Meeks and Yau solutions.

This is based on a joint work with Giovanni Bellettini (UNISI) and Riccardo Scala (UNISI).

Regularity of stationary varifolds with codimension at least 2

Paolo De Donato (UNIROMA1)

Almgren in his acclaimed work proved an estimate on the Hausdorff dimension of the singular set for area-minimizing currents with arbitrary codimension. One of the many tools he introduced in this work is the so-called frequency function, which has proven itself to be a powerful and versatile tool in order to approach several regularity problems in geometric measure theory. In this talk I present a joint work with prof. Emanuele Spadaro, in which we define a frequency function for stationary varifolds representable as the graph of a two-valued $C^{1,\alpha}$ function and use it to estimate the Minkowski content of the set of its singular branching points.

A unique continuation result for area minimizing currents

Stefano Decio (ETH)

Can two minimal surfaces touch each other to infinite order at a point without coinciding in a neighborhood of the point? Intuition from the theory of unique continuation for elliptic PDEs suggests this should not happen. Of course, part of the game here is to specify the notion of minimal surface. In joint work with Camillo Brena we give an answer to an instance of the question above: if an m-dimensional area minimizing integral current has infinite order of contact at a point with an m-dimensional surface with zero mean curvature then the current coincides with the surface in a neighborhood of the point.

Regularity for elliptic PDEs degenerating on lower dimensional manifolds

Gabriele Fioravanti (UNITO)

In this talk, we present some regularity results for elliptic PDEs involving singular or degenerate weights on a lower dimensional manifolds. The prototype of this equations is

$$-\mathrm{div}(|y|^a A(x,y)\nabla u(x,y)) = 0, \tag{1}$$

where $(x,y) \in \mathbb{R}^{d-n} \times \mathbb{R}^n$, with $n \geq 2$ and $a + n > 0$ (which means that $|y|^a \in L^1_{loc}(\mathbb{R}^d)$) and A is a uniformly elliptic matrix with ellipticity constant $0 < \lambda \leq \Lambda$. The place of *degeneration* of the coefficients, where the uniform ellipticity is lost, is the set $\{y = 0\}$, which has dimension $d - n$. Under suitable assumptions, we can prove $C^{0,\alpha}_{loc}$ and $C^{1,\alpha}_{loc}$ regularity of solutions to (1), for some *explicit* $\alpha = \alpha(n, a, \frac{\lambda}{\Lambda})$. The proof of our result is based on techniques such as approximation via perforated domains, blow-up analysis and Liouville-type Theorems.

This is a joint project with with Gabriele Cora (UNITO) and Stefano Vita (UNIPV).

Stability of multiphase mean curvature flow beyond a circular topology change

Alice Marveggio (Hausdorff Center of Mathematics)

The evolution of a network of interfaces by mean curvature flow features the occurrence of topology changes and geometric singularities. As a consequence, classical solution concepts for mean curvature flow are in general limited to a finite time horizon. At the same time, the evolution beyond topology changes can be described only in the framework of weak solution concepts (e.g., Brakke solutions), whose uniqueness may fail. Following the relative energy approach, we prove a quantitative stability estimate holding up to the singular time at which a circular closed curve shrinks to a point. This implies a weak-strong uniqueness principle for weak varifold-BV solutions to planar multiphase mean curvature flow beyond circular topology changes. We expect our method to have further applications to other types of shrinkers.

This talk is based on a joint work with Julian Fischer, Sebastian Hensel and Maximilian Moser.

Geometric and capacitary methods for Poincaré-Sobolev inequalities

Francesco Bozzola (UNIPR)

The inradius of an open set is a geometric quantity naturally linked to its sharp Poincaré-Sobolev embedding constants. In general, providing two-sided estimates for these quantities in terms of the inradius is not always possible, unless some geometric/topological or capacitary assumptions comes into play. This is due to a removability issue: removing sets with null capacity from a domain does not affect its Poincaré-Sobolev constants, but the inradius changes. Another possibility, would be that of "relaxing" the definition of inradius, by introducing a capacitary variant of it which is no more sensitive to the removal of sets with null capacity: the capacitary inradius. In this talk, we will explore both the possibilities by showing some new and classical results. This talk is based on a joint project with Lorenzo Brasco.
