BOOK OF ABSTRACTS

CALCULUS OF VARIATIONS AND FREE BOUNDARY PROBLEMS X

9 - 10 JANUARY 2025 - UNIVERSITÀ DI PISA

Spectral stability for Aharonov-Bohm operators with moving poles

Veronica Felli - Università di Milano-Bicocca

The behavior of eigenvalues of Aharonov-Bohm operators and their stability with respect to changes in the position of the poles are discussed. In particular, the problem of determining the exact asymptotic behavior of the eigenvalue variation, under small perturbations of the poles' configuration, is addressed.

Both the case of multiple colliding poles with circulation 1/2 and the case of any circulations are considered. In the case of half-integer circulations, a gauge transformation makes the problem equivalent to an eigenvalue problem for the Laplacian in a domain with straight cracks, laying along the directions of motion of the poles. For this problem, an asymptotic expansion for simple eigenvalues shows, as the dominant term, the minimum of an energy functional associated with the configuration of poles and defined on a space of functions suitably jumping through the cracks. In the case of operators with non-half-integer circulations, the problem is reformulated as a system of two equations (with unknowns given by the real and imaginary parts of the gauged eigenfunction) coupled through prescribed jumps of the unknown functions and their normal derivatives on some segments identified by the configuration of the poles.

In the case of a single moving pole, double eigenvalues are also considered: it is observed that they split into two branches of simple eigenvalues as the pole moves along certain directions.

The results presented in the talk have been obtained in a series of papers in collaboration with L. Abatangelo, B. Noris, R. Ognibene, and G. Siclari.

Variational worn stones

Ilaria Fragalà - Politecnico di Milano

We discuss an evolution model à la Firey for a convex stone which undertakes an energy based erosion process. Under the assumption of existence of a solution to the corresponding parabolic flow, we show that the stone tends to become asymptotically spherical, relying on new symmetry results for overdetermined boundary value problems.

Based on a joint work with Graziano Crasta (Università La Sapienza, Roma).

Sharp bounds on the Nusselt number in Rayleigh-Bénard convection

Andrea Malchiodi - Scuola Normale Superiore

We analyze the problem in fluid dynamics of deriving bounds for heat transportation in the infinite Prandtl number limit. Due to a maximum principle property for the temperature, this amounts to proving a-priori bounds for horizontally-periodic solutions of a fourth-order equation in a strip of large width with both Dirichlet and Neumann data. We obtain such bounds using Fourier analysis, integral representations, and a bilinear estimate due to Coifman and Meyer which uses the Carleson measure characterization of BMO functions by Fefferman.

This is joint work with S. Chanillo.

Partial regularity in nonlocal problems

Giuseppe Mingione - Università di Parma

The theory of partial regularity for elliptic systems replaces the classical De Giorgi-Nash-Moser one for scalar equations asserting that solutions are regular outside a negligible closed subset called the singular set. Eventually, Hausdorff dimension estimates on such a set can be given. The singular set is in general non-empty. The theory is classical, started by Giusti & Miranda and Morrey, in turn relying on De Giorgi's seminal ideas for minimal surfaces.

I shall present a few results aimed at extending the classical local partial regularity theory to nonlinear integrodifferential systems and to provide a few basic, general tools in order to prove so called epsilon-regularity theorems in general non-local settings. From recent, joint work with Cristiana De Filippis (Parma) and Simon Nowak (Bielefeld).

Singularly perturbed elliptic systems modeling partial separation and their free boundaries

Susanna Terracini - Università di Torino

We investigate the asymptotic behavior, as $\beta \to +\infty$, of solutions to competition-diffusion system of type

$$\begin{cases} \Delta u_{i,\beta} = \beta u_{i,\beta} \prod_{j\neq i} u_{j,\beta}^2 & \text{in } \Omega, \\ u_{i,\beta} = \varphi_i \geq 0 & \text{on } \partial\Omega, \end{cases} \qquad i = 1, 2, 3,$$

where $\varphi_i \in W^{1,\infty}(\Omega)$ satisfy the *partial segregation condition*

$$\varphi_1 \, \varphi_2 \, \varphi_3 \equiv 0 \quad \text{in } \overline{\Omega}.$$

For $\beta > 1$ fixed, a solutions can be obtained as a minimizer of the functional

$$J_\beta(\mathbf{u}, \Omega) := \int_\Omega \Big(\sum_{i=1}^3 |\nabla u_i|^2 + \beta \prod_{j=1}^3 u_j^2 \Big) \, dx$$

on the set of functions in $H^1(\Omega, \mathbb{R}^3)$ with fixed traces on $\partial\Omega$. We prove *a priori* and *uniform in* β Hölder bounds. In the limit, we are lead to minimize the energy

$$J(\mathbf{u}, \Omega) := \int_\Omega \sum_{i=1}^{3} |\nabla u_i|^2 \, dx$$

over all partially segregated states:

$$u_1 \, u_2 \, u_3 \equiv 0 \quad \text{in } \overline{\Omega}$$

satisfying the given, partially segregated, boundary conditions above. We prove regularity of the free boundary up to a low-dimensional singular set.

This is based on joint works with Nicola Soave (Università di Torino).

Isoperimetric Inequalities via the ABP method

Giona Veronelli - Università di Milano-Bicocca

In the 1960s, A.D. Alexandroff, I.G. Bakelman, and C. Pucci proved a celebrated maximum principle, introducing a technique that inspired the so-called ABP method. In this talk, we will survey some applications of the ABP method to prove isoperimetric inequalities, starting with the elegant new proof of the Euclidean isoperimetric inequality by X. Cabré and highlighting the more recent breakthrough generalization by S. Brendle to Riemannian manifolds with non-negative curvature and their minimal submanifolds.

In the final part of the talk, we will discuss recent results obtained in collaboration with D. Impera, M. Rimoldi, and S. Pigola, where a certain amount of negative curvature - either localized in space or small in mass - is permitted.