Book of abstracts

Regularity theory for free boundary and geometric variational problems II

Pisa (Italy), 11-15 July 2022

Construction of singular minimal hypersurfaces

Leon Simon

Discussion of recent work on the construction of singular minimal hypersurfaces including (1) examples which, with respect to a smooth ambient metric in $\mathbb{R}^n \times \mathbb{R}^m$, are stable embedded and have singular set an arbitrary closed set in $\{0\} \times \mathbb{R}^m$, and (2) examples which, with respect to the Euclidean metric for $\mathbb{R}^n \times \mathbb{R}^m$, have an isolated singularity at 0 and cylindrical tangent cone $C \times \mathbb{R}^m$.

A mesoscale flatness criterion and its application to exterior isoperimetry

Francesco Maggi

We introduce a "mesoscale flatness criterion" for hypersurfaces with bounded mean curvature, discussing its relation and its differences with classical blow-up and blow-down theorems, and then we exploit this tool for a complete resolution of relative isoperimetric sets with large volume in the exterior of a compact obstacle. This is joint work with Michael Novack at UT Austin.

Non-degenerate minimal surfaces as energy concentration sets: a variational approach

Guido De Philippis

I will show that every non-degenerate minimal sub-manifold of codimension 2 can be obtained as the energy concentration set of a family of critical points of the (rescaled) Ginzburg Landau functional. The proof is purely varia-tional, and follows the strategy laid by Jerrard and Sternberg in 2009. The same proof applies also to the Yang-Mills-Higgs and to the Allen-Cahn-Hillard energies. This is a joint work with Alessandro Pigati.

Higher codimension area-minimizers: understanding interior branch points via the frequency function

Anna Skorobogatova

Integral currents provide a natural setting in which to study the Plateau problem, but permit the formation of singularities in area-minimizers. The problem of determining the size and structure of the interior singular set of an area-minimizer in this setting has been studied by many since the 1960s, with many ground-breaking contributions. When the codimension is higher than 1, due to the presence of singular points with flat tangent cones, little progress has been made since the celebrated $(m-2)$-Hausdorff dimension bound on the singular set due to Almgren, the proof of which has since been simplified by De Lellis and Spadaro.

In this talk I will discuss joint work in progress with Camillo De Lellis towards getting a more refined characterization of the behaviour of the surface at singular points with flat tangent cones, including uniqueness of the flat tangent cone in certain cases, as well as other interesting properties.

Regularity for the mean curvature flow with boundary

Carlo Gasparetto

We discuss an ε-regularity theorem for Brakke flows with a prescribed boundary. The proof we present is based on viscosity techniques introduced by O. Savin in the context of elliptic equations.

A free boundary perspective on transmission and inverse scattering problems

Henrik Shahgholian

In this talk, I will present some free boundary aspects of transmission and inverse scattering problems. I shall focus on questions related to the regularity theory of solutions and the unknown intefaces, that appear in these problems. The talk will be kept at a narrative and heuristic level.

Regularity (quantitative convexity) of the boundary for subsets of a convex domain that almost minimize the perimeter

Guy David

I'll try to describe joint work with D. Jerison that concerns the boundary of subsets of a convex domain (for instance) that almost minimize the perimeter (or minimize it under a volume constraint). I'll try to describe the local connectedness of the (simplified reduced) boundary, for instance measure by the fact that the geodesic distance there is locally equivalent to the ambient (Euclidean) distance.

Contact points with integer and 7/2 frequencies in the thin obstacle problem

Hui Yu

The thin obstacle problem is a classical free boundary problem arising from the study of an elastic membrane resting on a lower-dimensional obstacle. Concerning the behavior of the solution near a contact point between the membrane and the obstacle, many important questions remain open.

In this talk, we discuss a unified method that leads to a rate of convergence to "tangent cones" at contact points with integer frequencies in general dimensions as well as 7/2-frequency points in 3d.

This talk is based on recent joint works with Ovidiu Savin (Columbia).

Generic uniqueness for optimal transportation networks

Simone Steinbrüchel

In this talk, we consider optimal transport in the setting of normal 1-currents in Euclidean space. For $0 < \alpha < 1$ and b a boundary, we minimize the α-mass among all normal currents with boundary b. Already when the boundary is supported on four points, one can construct examples with two different minimizers for the same boundary. However, I will present a constructive proof that for the generic boundary, in the sense of Baire categories, the minimizer is unique. This is a joint work with G. Caldini and A. Marchese.

Free boundary partial regularity in the thin obstacle problem

Federico Franceschini

The Thin Obstacle Problem is a classical free boundary problem to which many techniques coming from Minimal surfaces can be applied. In particular, Almgren's frequency function is monotone, so to each free boundary point a certain "frequency" can be attached. The classification of (necessarily homogeneous) blow-ups is only known in 2D. After a general introduction, I will present a recent work with Joaquim Serra in which we prove, in every dimension, existence of a 2D blowup with a Lipschitz rate of convergence, at least outside of a co-dimension 3 set in the ambient space. I will discuss also some enhancement of this result (still work in progress) .

Initial stability estimates for Ricci flow and three dimensional Ricci-pinched manifolds

Felix Schulze

We investigate the question of stability for a class of Ricci flows which start at possibly non-smooth metric spaces. We show that if the initial metric space is Reifenberg and locally bi-Lipschitz to Euclidean space, then two solutions to the Ricci flow whose Ricci curvature is uniformly bounded from below and whose curvature is bounded by ct^{-1} converge to one another at an exponential rate once they have been appropriately gauged. As an application, we show that smooth three dimensional, complete, uniformly Ricci-pinched Riemannian manifolds with bounded curvature are either compact or flat, thus confirming a conjecture of Hamilton and Lott. This is joint work with A. Deruelle and M. Simon.

Network flow: resolution of singularities and stability

Alessandra Pluda

The curve shortening flow is an evolution equation in which a curve moves with normalvelocity equal to its curvature, and can be interpreted as the gradient flow of the length.In this talk we consider the same flow for networks (finite unions of sufficiently smooth curveswhose endpoints meet at junctions). We will explain how to define the flow in a classical PDE framework, and then we will list some examples of singularity formation, both at finite andinfinite time, and explain the resolution of such singularities obtained by geometric microlocalanalysis techniques. Finally we will show how starting from a suitable Lojasiewicz-Simon in-equality it is possible to prove the stability of the flow in the sense that a network sufficientlyclose in the H^2-norm to a minimal one exists for all times and converges smoothly.

This seminar is mainly based on two recent papers in collaboration with Jorge Lira (Universidade Federal do Ceará), Rafe Mazzeo (Stanford University), Mariel Saez (P. Universidad Catolica de Chile) and Marco Pozzetta (Università di Napoli Federico II).

Area minimizing hypersurfaces modulo p: regularity and structure of singular sets

Salvatore Stuvard

Integer rectifiable currents modulo p (for an integer $p \geq 2$) are a class of generalized surfaces in which it is possible to define and solve Plateau's problem. Minimizers modulo p exhibit a far richer geometric complexity than minimizers in the classical sense of integral currents, and the corresponding singular structures are often observed in soap films. In this talk I will present some recent results concerning the partial regularity of area minimizing currents modulo p and the structure of their singular sets, which emphasis on the codimension-one case. Based on joint works with Camillo De Lellis (IAS), Jonas Hirsch (U Leipzig), Andrea Marchese (U Trento), and Luca Spolaor (UCSD).

A strong maximum principle for minimizers of the one-phase Bernoulli problem

Nick Edelen

We prove a strong maximum principle for minimizers of the one-phase Alt-Caffarelli functional. We use this to construct a Hardt-Simon-type foliation associated to any 1-homogenous global minimizer.

The multi-bubble problems for convex sets

Riccardo Tione

The k-bubble problem concerns the classification of minimizers/critical points of the area of the boundary of k sets under variations that keep the volume of every set fixed. For $k = 1$, one obtains the classical isoperimetric problem. In this case, one is led to study constant mean curvature surfaces. Under mild assumptions on the surface, it can be shown that the only solution is then a sphere. If $k = 2$ M. Hutchings, F. Morgan, M. Ritoré and A. Ros showed that the only minimizer for the problem in $\mathbb{R}^3$ is the standard double bubble. Very recently, E. Milman and J. Neeman proved that the only minimizer in $\mathbb{R}^3$ for $k = 3$ is the standard triple bubble. In this talk, I will present a work in progress with A. De Rosa concerning the classification of critical points for the double and triple bubble ($k = 2$ and $k = 3$) problem assuming the sets involved to be convex.

On the De Giorgi-Nash-Moser theorem for hypoelliptic operators

Jonas Hirsch

I would like to present a relative simple approach to show uniform boundedness and a weak Harnack inequality for general hypoelliptic operators. The novelty is the avoidance of a "general" Sobolev embedding and a "quantitative" Poincare inequality. Somehow our approach shows that one can somehow consider even the classical De Giorgi-Nash-Moser theorem as a "perturbation" of the poisson equation. If time permits I would like to discuss as well how the geometry of the hypoelliptic equations come into play to obtain as a consequence the famous Hölder regularity.

The presented results are work in progress with Helge Dietert.

Free discontinuity problems in Stokes fluids

Mickaël Nahon

We consider an incompressible Stokes fluid contained in a box B that flows around an obstacle $K \subset B$ with a Navier boundary condition on ∂K. I will present existence and partial regularity results for the minimization of the drag of K among all obstacles of given volume.

Branched regularity theorems for stable codimension one stationary integral varifolds near higher multiplicity cones

Paul Minter

In this talk we will discuss some regularity results analysing the behaviour of a large class of stable, codimension one, stationary integral varifolds near certain singular points which have tangent cones occurring with higher multiplicity. The two cases considered are when one tangent cone is a higher multiplicity hyperplane (as is the case at a branch point), or a (non-flat) union of half-hyperplanes meeting along a common axis, with each half-hyperplane occurring with some multiplicity. Understanding singularities with tangent cones of higher multiplicity, in particular branch points, is a key difficulty in understanding the structure and regularity of minimal hypersurfaces constructed in various settings. In both our cases, we are able to establish that not only is the tangent cone unique, but that a certain multi-valued $C^{1,\alpha}$ graph structure holds locally about the singular point. One application of our results is to understanding the structure of flat singular points in codimension one area minimising currents mod p. Some results are joint with Neshan Wickramasekera.

Optimal regularity for the fully nonlinear thin obstacle problem

Xavier Fernández-Real

The thin obstacle problem for an elliptic operator L with zero obstacle is

$$\begin{cases} Lu &= 0 \quad \text{in } B_1 \setminus \{x_n = 0, u = 0\} \\ Lu &\leq 0 \quad \text{in } B_1 \\ u &\geq 0 \quad \text{on } B_1 \cap \{x_n = 0\}. \end{cases} \tag{1}$$

When L is the Laplacian, this is also called the Signorini problem, and it is now a classical free boundary problem. In this case, the $C^{1,\alpha}$ (one-sided) regularity of solutions was first proved by Caffarelli in 1979, and it was not until 2004 that Athanasopoulos and Caffarelli were able to prove the optimal regularity of solutions, $C^{1,1/2}$. This was done by observing that solutions to the thin obstacle problem satisfy certain monotonicity formulas, available only because $L = \Delta$.

In this talk we deal with the nonlinear generalization consisting in taking $Lu = F(D^2u)$ for some convex uniformly elliptic fully nonlinear operator F.

Under such conditions, the $C^{1,\alpha}$ regularity was proved in 2016, and then a study of regular points was performed in 2017 by Ros-Oton and Serra, where they are locally a C^1 manifold. Nonetheless, due to the lack of monotonicity formulas, the optimal regularity of solutions was still not known.

In this talk, we present our recent results together with M. Colombo and X. Ros-Oton, where we show what the optimal regularity of (1) is. In particular, we prove that, if in addition F is rotationally invariant, then solutions are always $C^{1,1/2}$.

Min-max construction of anisotropic CMC surfaces

Antonio De Rosa

We prove the existence of nontrivial closed surfaces with constant anisotropic mean curvature with respect to elliptic integrands in closed smooth 3-dimensional Riemannian manifolds. The constructed min-max surfaces are smooth with at most one singular point. The constant anisotropic mean curvature can be fixed to be any real number. In particular, we partially solve a conjecture of Allard [Invent. Math.,1983] in dimension 3. Joint work with Guido De Philippis.

Branch points for (almost-)minimizers of two-phase free boundary problems

Mariana Smit Vega Garcia

In this talk, we will discuss minimizers and almost-minimizers of Alt-Caffarelli-Friedman type functionals. In particular, we will consider branch points in their free boundary. This is based on recent joint work with Guy David, Max Engelstein, and Tatiana Toro.
